International Journal of Theoretical Physics, Vol. 45, No. 11, November 2006 (© 2006)
DOI: 10.1007/s10773-006-9168-2

New Electroweak Formulation Fundamentally Accounting for the Effect Known as "Maximal Parity-Violation"

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Received December 12, 2005; Accepted March 20, 2006
Published Online: June 6, 2006

The electroweak scheme is wholly recast, in the framework of a relativistic quantum field formalism being a covariant fermion–antifermion extension of the usual one for massive spin-$\frac{1}{2}$ point fermions. The new formalism is able to reread the "maximal P-violation" effect in a way restoring P and C symmetries themselves: it provides a natural "chiral field" approach, which gives evidence of the existence of a pseudoscalar (extra) charge variety anticommuting with the scalar (ordinary) one and just underlying the "maximally P-violating" phenomenology. Its zero-mass limit leads to a strict "chiral" particle theory, which remodels any massless spin-$\frac{1}{2}$ fermion and corresponding antifermion as two mere pseudoscalar-charge eigenstates being the simple mirror images of each other. On such a basis, the (zero-mass) electroweak primary fermions are all redefined to be (only left-handed) "chiral" particles (with right-handed complements just standing for their antiparticles) and to carry at most scalar charges subjected as yet to a maximal uncertainty in sign: it is only by acquiring mass, and by gaining an extra helicity freedom degree, that they now may also manifest themselves as "Dirac" particles, with sharp scalar-charge eigenvalues. The fermion-mass appearance is thus made herein a dynamical condition strictly necessary to obtain actual superselected scalar-charge (and first, electric-charge) eigenstates. A pure "internal" mass-generating mechanism, relying only on would-be-Goldstone bosons (even to yield fermion masses) and no longer including an "external" Higgs contribution, is adopted accordingly. This is shown to be a self-consistent mechanism, which still maintains both renormalizability and unitarity. It involves a P-breaking in the neutral-weak-current sector (due to the Weinberg mixing) while it leaves the charged-current couplings truly P-invariant even in the presence of a (standardly parametrized) CP-violation.

KEY WORDS: origin of "maximally parity-violating" phenomenology.

1. INTRODUCTION

Recently a new theoretic approach to the electroweak scheme (Glashow, 1961; Salam, 1968; Weinberg, 1967) has been suggested, which leads indeed to a *spontaneous* prediction of the "maximal parity-violation" effect (Ambler

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et al., 1957; Lee and Yang, 1956), without need any more (Ross, 1993; Narlikar and Padmanabhan, 1986) of *ad hoc* inputs such as the "$V - A$" prescription (Feynman and Gell-Mann, 1958; Marshak and Sudarshan, 1958; Sakurai, 1958) and the neutrino two-component model (Landau, 1957; Lee and Yang, 1957; Salam, 1957; Weyl, 1929). It starts from a strictly covariant fermion–antifermion generalization (Ziino, 1996) of the usual quantum field formalism for nonzero-mass fermions; and it has been applied, as a first step, to the leptonic sector (Ziino, 2000).

The generalized basic formalism just mentioned can naturally deal not only with Dirac fields, as eigenfields of a *scalar* (ordinary) charge variety, but also with *true* "chiral fields," as eigenfields of a *pseudoscalar* (extra) charge variety (the term "charge" is here used, of course, in a broad sense, to signify any additive internal quantum number). The mutual property peculiar to such varieties of charges is the fact that they *anticommute*. In this enlarged framework, one may define, on the same footing, both a covariant pair of "scalar-charge conjugated" Dirac fields and a covariant pair of "pseudoscalar-charge conjugated" chiral fields, and one may globally think of a *dual* (either "Dirac" or "chiral") massive fermion–antifermion model, characterized by the actual coexistence of both (scalar and pseudoscalar) kinds of charges and by the corresponding dynamical alternation of two distinct pairs of *superselected* charge-conjugated eigenstates. A model like this, formally supported by the replacement of the (ad hoc) Dirac-field "$V - A$" current with a (natural) chiral-field "V" current, can not only account for the "maximally P-violating" phenomenology, but even recover (paradoxically) P and C individual symmetries: it enables one to reinterpret the well-known CP mirror symmetry of the "$V - A$" formalism as just *a P mirror symmetry between fermions and antifermions in their alternate showing as net pseudoscalar-charge, rather than scalar-charge, conjugated eigenstates*. The extreme consequences of such a view can be drawn by taking the zero-mass limit: thus, one is automatically left with a *pure and simple* "chiral" fermion–antifermion model, which intrinsically redefines whatever massless spin-$\frac{1}{2}$ point particle and its antiparticle as two *permanent* (left- and right-handed) pseudoscalar-charge eigenstates, each just coinciding with the *ordinary* (merely helicity-conjugate) *mirror image* of the other.

The present paper tries to extend the new approach to the whole electroweak scheme (including the quark sector). The overall formulation here proposed (containing some ameliorations and adjustments, as compared with the previous one for the only leptonic sector) gives also a much more careful account of the far-reaching effects on the Higgs-boson question (Ziino, 2003). For the paper to be made as self-contained as possible, the next three sections are just devoted to a review of the underlying formalism.

2. NATURAL THEORETICAL GROUNDS FOR A "MAXIMALLY *P*-VIOLATING" PHENOMENOLOGY, AND A NEW FORMAL READING OF IT WITH BOTH *P* AND *C* SYMMETRIES PARADOXICALLY RECOVERED

It is well-known that the Dirac quantum field formalism cannot provide a one-particle relativistic description: the associated Fock space is necessarily the sum of two pure positive-energy Fock spaces – referring (in Dirac's language) to "particles" and "holes" respectively – which are taken into each other by a suitable operation of "particle"$\rightleftharpoons$"hole" conjugation. It is also well-known that a (manifestly covariant) one-particle description may all the same be restored, once the Stüeckelberg–Feynman general approach to the negative-energy problem is adopted (Aitchison and Hey, 1984; Feynman, 1949; Stüeckelberg, 1948): the "hole" motion (forwards in time) can then be reread as a negative-energy "particle" motion, *backwards* in time, and the Fock space above can likewise be recast as a *single* one for "particles" only, with energies now covariantly running over the *entire* spectrum of positive and negative eigenvalues. Thanks to this improved view (valid for both fermions and bosons) the "particle–hole" language has clearly lost its original motivations; yet, the use of such a language may still turn out convenient, if one wants to make somehow a distinction between a "fermion" picture (in which, i.e., one has "particle" = fermion and "hole" = antifermion) and an "antifermion" picture (in which, conversely, one has "particle" = antifermion and "hole" = fermion).

Let $\mathcal{F}^\circ$ denote the above (manifestly covariant) Stüeckelberg–Feynman Fock space, and let it stand in particular for a "fermion" Fock space (where, i.e., "particle" = fermion). This definition of $\mathcal{F}^\circ$ might lead one to wonder whether a *covariant* charge-conjugation operation can be introduced too, which may be able to turn $\mathcal{F}^\circ$ into another space being an "antifermion" Fock space (where, i.e., "particle" = antifermion). Such an operation should have the effect of *transforming a fermion taken in its whole* (positive- and negative-energy) *spectrum, into the corresponding* (positive- and negative-energy) *antifermion*; it should then be, in principle, *not* just the same as the "particle" $\rightleftharpoons$ "hole" conjugation (which, on the contrary, is a mere *non*covariant operation interchanging positive-definite-energy objects). The trouble is, however, that $\mathcal{F}^\circ$ can *itself* be reinterpreted as an "antifermion" Fock space, since in the Stüeckelberg–Feynman view a complete set of $\mathcal{F}^\circ$ kets (bras) for fermions will clearly amount to a complete set of $\mathcal{F}^\circ$ bras (kets) for antifermions. So, at first sight, trying to define a covariant charge-conjugation operation would seem to be a trivial matter, as we cannot think of any *further* Fock space being the "covariant charge-conjugate" of $\mathcal{F}^\circ$. This just corresponds to the fact that, in a symmetrized "particle"–"hole" standard description, one may indifferently put *either* "particle" = fermion (and "hole' = antifermion) *or* "particle" = antifermion (and "hole" = fermion).

Actually, the question can be set anew with the aid of a careful re-examination of the Stüeckelberg–Feynman approach from a classical viewpoint. Let $-p^{\mu} = m(-u^{\mu})$ ($\mu = 0, 1, 2, 3$; metric: $+---$) be the four-momentum of a negative-energy particle of proper (i.e., covariant) mass m (> 0) and four-velocity $-u^{\mu} = -dx^{\mu}/ds$ ($-dx^0 < 0$). Since the equivalent positive-energy antiparticle, with four-momentum p^{μ}, is just moving along the same world-line in the *opposite* direction, $ds \to -ds$, one has that the "slope" $-u^{\mu}$ of that world-line will be left *unchanged* by the Stüeckelberg–Feynman procedure, $(-dx^{\mu})/ds = dx^{\mu}/(-ds)$. Strictly speaking, it should therefore be claimed that in replacing $-p^{\mu}$ (for the particle) with p^{μ} (for the antiparticle) a pure change of the *proper-mass sign* is involved, $-p^{\mu} \to p^{\mu} \Longrightarrow m \to -m$. This does not seem to be a very surprising result: due to the *quadratic* character of the energy–momentum relation $E^2 = \mathbf{p}^2 + m^2$ ($c = 1$), we may clearly associate both energy roots $\pm E$ with the same (positive) proper mass m, but we may just as well associate both proper-mass roots $\pm m$ with the same (positive) energy E. The fact is that the relative sign of energy and proper mass does depend on either sign of the four-velocity time component $\pm u^0$. So, the assignment of a proper mass $-m$ to the antiparticle cannot even be said to clash with the *CPT* theorem: what *rigorously* follows from the validity of *CPT* symmetry is that a real particle and its antiparticle must have identical *rest energies*, which does only mean that m^2, and not m itself, must be equal for them both. If these arguments are in particular applied to fermions, it may then be stated that a free Dirac fermion and the associated antifermion can *covariantly* be distinguished by the (opposite) signs of their proper masses. This just enables one to think of a *non*trivial "covariant charge-conjugation" operation for free fermions: it should coincide with the pure internal operation of *proper-mass reversal* (Costa de Beauregard, 1982, 1984; Recami and Ziino, 1976; Sakurai, 1958; Tiomno, 1955) (leaving both four-momentum and helicity unvaried). It is evident, on the other hand, that the proper-mass sign in the Dirac equation is immaterial and has no observable kinematical effects. The only problem is that the Fock space $\mathcal{F}^{\circ}$ should clearly be left structured in such a way that it may still *equally* refer to (positive- and negative-energy) fermions or antifermions. This requirement can be maintained, only if the (positive- and negative-energy) $\mathcal{F}^{\circ}$ states are assumed to depend merely on $|m|$ (just in line with the fact that they should always be completely specified by four-momentum and helicity). One is thus led to conclude that the *strict* Fock space $\mathcal{F}^{\circ}$ does not appear to be adequate for allowing an explicit definition of proper-mass reversal as a "covariant charge-conjugation" operation. What would be needed is some "enlarged" Fock space which may be able as well to make a *manifest* formal distinction between a "fermion" (positive- and negative-energy) covariant Dirac picture, marked by the m^2 root $+m$, and an "antifermion" one, marked by the m^2 root $-m$. In other words, we should find the way to *double* $\mathcal{F}^{\circ}$, by giving it some "label" that may unambiguously tell us which of the two alternative complete pictures above is being considered.

To this aim, it turns out appropriate to introduce two (orthogonal) unit internal state-vectors, $|\, f\rangle$ and $|\, \bar{f}\,\rangle$, defined as eigenvectors of a (one-particle) proper-mass operator, M, with eigenvalues $+m$ and $-m$:

$$M|\, f\rangle = +m\,|\, f\rangle, \quad M|\, \bar{f}\,\rangle = -m\,|\, \bar{f}\,\rangle. \tag{1}$$

Let S_{in} be the two-dimensional (internal) space which is spanned by such eigenvectors. We may then say that a "dressed" Fock space $\mathcal{F}$ can be built from the "bare" one $\mathcal{F}^{\circ}$, being such that

$$\mathcal{F} = \mathcal{F}^{\circ} \otimes S_{\text{in}}. \tag{2}$$

As an effect of a "dressing" procedure like this, the original complete set of $\mathcal{F}^{\circ}$ kets (bras) has indeed been turned into a "Dirac fermion" set, covariantly labelled by $|\, f\rangle$ ($\langle f\,|$), *plus* a "Dirac antifermion" one, covariantly labelled by $|\, \bar{f}\,\rangle$ ($\langle \bar{f}\,|$), with an energy range still including, in either case, both positive and negative eigenvalues. Evidently, $\mathcal{F}$ will contain two distinct ("fermionic" and "antifermionic") vacuum states, $|0\rangle|\, f\rangle$ and $|0\rangle|\, \bar{f}\,\rangle$, instead of a single ("bare") covariant fermion–antifermion vacuum state, $|0\rangle$; this, of course, does not prevent us from constructing suitable "undressing" annihilation operators (multiplied by $\langle f\,|$ or $\langle \bar{f}\,|$) and corresponding "dressing" creation operators (multiplied by $|\, f\rangle$ or $|\, \bar{f}\,\rangle$) which may connect $\mathcal{F}$ with the (one-dimensional) space spanned by $|0\rangle$ (note, however, that the *strict* annihilation and creation operators, subject to standard anticommutation rules, are always left those defined in the *strict* Fock space $\mathcal{F}^{\circ}$). Going over to $\mathcal{F}$, we may rigorously think of a "covariant charge-conjugation" as being accomplished by a unitary and Hermitian operator, C_{cov}, which properly acts within S_{in} and trivially behaves (just like an identity operator) within $\mathcal{F}^{\circ}$:

$$C_{\text{cov}}|\, f\rangle = |\, \bar{f}\,\rangle, \quad C_{\text{cov}}|\, \bar{f}\,\rangle = |\, f\rangle \quad \left(C_{\text{cov}}^{-1} = C_{\text{cov}}^{\dagger} = C_{\text{cov}}\right). \tag{3}$$

We can also see that C_{cov} anticommutes with M, in line with the fact that it primarily works as a *proper-mass conjugation* operator. By the way, the choice of a self-explanatory symbol like C_{cov} is just aimed at avoiding that "covariant charge-conjugation" may be confused with the usual (noncovariant) "particle" $\rightleftharpoons$ "hole" conjugation: the latter one will be still denoted by the ordinary symbol C.

The Fock-space *doubling* brought in by (2) has a meaning that can be conveniently expressed in the "particle–hole" language: it leads to a generalized description which distinctly includes both a *fermionic* covariant Dirac picture, with "particle" = fermion and "hole" = antifermion, and an (equally admissible) *antifermionic* one, with "particle" = antifermion and "hole" = fermion. These pictures – which may be said to be "covariantly conjugated" to each other – are just distinguished by either sign of proper mass. So, a pair of Dirac free-field equations like

$$i\gamma^{\mu}\partial_{\mu}\psi_{f} = +m\,\psi_{f}, \quad i\gamma^{\mu}\partial_{\mu}\psi_{\bar{f}} = -m\,\psi_{\bar{f}} \tag{4}$$

($\hbar = c = 1$; $\gamma^{0\dagger} = \gamma^{0}, \gamma^{k\dagger} = -\gamma^{k}, k = 1, 2, 3$) is to be associated with them, where $\psi_{\bar{f}}$ should consistently stand for the *proper-mass conjugate* counterpart of ψ_{f}. Let $u_{f}(\mathbf{p})$ and $u_{\bar{f}}(\mathbf{p})$ be accordingly two positive-energy eigenspinors satisfying the equations

$$\gamma^{\mu}p_{\mu}u_{f} = +m\,u_{f}, \quad \gamma^{\mu}p_{\mu}u_{\bar{f}} = -m\,u_{\bar{f}}. \tag{5}$$

For $\mathbf{p} = 0$, one clearly obtains

$$\gamma^{0}\,(+m)\,u_{f}(0) = E\,u_{f}(0)\;, \qquad \gamma^{0}\,(-m)\,u_{\bar{f}}(0) = E\,u_{\bar{f}}(0), \tag{6}$$

where E (> 0) is the related energy eigenvalue, such that $E = m$. By making this substitution, it is immediate then to see that

$$\gamma^{0}\,u_{f}(0) = u_{f}(0), \qquad \gamma^{0}\,u_{\bar{f}}(0) = -u_{\bar{f}}(0). \tag{7}$$

One therefore has that due to the discordant signs of the associated proper masses, the opposite-intrinsic-parity requirement for the eigenspinors $u_{f}(\mathbf{p})$ and $u_{\bar{f}}(\mathbf{p})$ can be automatically fulfilled by the application of *one and the same* parity matrix (say, γ^{0}) to them both. This is not the case of two mere "particle" and "hole" conjugated eigenspinors, which are coincident solutions of just either one of Eqs. (5) and then require two *discordant* parity representations (say, γ^{0} and $-\gamma^{0}$, respectively) for them to be assigned opposite intrinsic parities. As both of the field equations (4) are in turn compatible with the "bare" Fock space $\mathcal{F}^{\circ}$, we may build on the whole a *double*-structured, "undressing" field operator, $\Psi(x)$ ($x \equiv x^{\mu}$), which just reduces to $\psi_{f}(x)$ or $\psi_{\bar{f}}(x)$ according to whether applied to $\mathcal{F}^{\circ}$ states that are coupled to $|\,f\rangle$ or $|\,\bar{f}\,\rangle$: it looks like

$$\Psi(x) = \psi_{f}(x)\,\langle\, f\,| \,+\, \psi_{\bar{f}}(x)\,\langle\,\bar{f}\,|, \tag{8}$$

and obeys the *generalized Dirac equation*

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = \Psi(x)M, \tag{9}$$

M being the proper-mass operator defined by (1). This can be strictly said to be a *covariant fermion–antifermion field*. Besides being still a Lorentz four-spinor, it is also a (bra) *vector* in the internal space S_{in}, with $\psi_{f}(x)$ and $\psi_{\bar{f}}(x)$ correspondingly acting as its *components* relative to the orthonormal basis $\big(\langle\, f\,|, \langle\,\bar{f}\,|\big)$. Let $\psi_{f}(x)$ and $\psi_{\bar{f}}(x)$ be in particular called the "Dirac" (fermion and antifermion) components of $\Psi(x)$ in S_{in}, the former annihilating (either positive or negative energy) "Dirac" fermion states covariantly marked by $|\,f\rangle$, and the latter annihilating (either positive or negative energy) "Dirac" antifermion states covariantly marked by $|\,\bar{f}\,\rangle$. If we take account of (3), we can also define the C_{cov} counterpart of $\Psi(x)$ as

$$\Psi^{(C_{\rm cov})}(x) \equiv \Psi(x)C_{\rm cov} = \psi_f(x)\,\langle\bar{f}|\,+\,\psi_{\bar{f}}(x)\,\langle f| \tag{10}$$

and introduce the adjoints of Ψ and $\Psi^{(C_{\rm cov})}$, such that

$$\bar{\Psi}(x) = |f\rangle\,\bar{\psi}_f(x)\,+\,|\bar{f}\rangle\,\bar{\psi}_{\bar{f}}(x) \tag{11}$$

($\bar{\psi} = \psi^\dagger\gamma^0$) and $\bar{\Psi}^{(C_{\rm cov})}(x) \equiv C^\dagger_{\rm cov}\bar{\Psi}(x)$. From a comparison, e.g., of (10) with (8), it is immediate to realize that applying $C_{\rm cov}$ to both $\Psi(x)$ and $\bar{\Psi}(x)$ can equivalently be implemented by prescribing

$$C_{\rm cov} : \psi_f(x) \rightleftharpoons \psi_{\bar{f}}(x), \quad \bar{\psi}_f(x) \rightleftharpoons \bar{\psi}_{\bar{f}}(x). \tag{12}$$

This just leads us to state that $\psi_{\bar{f}}(x)$ should be *covariantly* obtained from $\psi_f(x)$ by simply demanding the proper-mass reversal $m \to -m$ in the Dirac equation obeyed by $\psi_f(x)$. So, after all, one may write (apart from a phase factor):

$$\psi_{\bar{f}}(x) = \gamma^5\psi_f(x), \quad \bar{\psi}_{\bar{f}}(x) = -\bar{\psi}_f(x)\gamma^5 \tag{13}$$

($\bar{\psi} = \psi^\dagger\gamma^0$; $\gamma^5 \equiv i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$). Such an outcome clearly calls for some explanatory comments. In view of (13)—and in accordance with the fact that $C_{\rm cov}$ is essentially defined in $S_{\rm in}$—one has that the Fourier expansions of $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ will share a *unique* type of "particle" annihilation operators, say, $a(\mathbf{p},\sigma)$, as well as a *unique* type of "hole" creation operators, say, $a^{\rm h\dagger}(\mathbf{p},\sigma)$ (σ being the helicity variable). This appears to be admissible, for the simple reason that $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ belong to two *alternative*, self-contained pictures – marked by $|f\rangle$ and $|\bar{f}\rangle$, respectively – each being as able as the other to describe the creation or annihilation of a "particle"–"hole" pair (even though with an interchange of the actual physical objects referred to as "particle" and "hole"): according to whether $\psi_f(x)\,\langle f|$ or $\psi_{\bar{f}}(x)\,\langle\bar{f}|$ is in turn involved, the same "particle" annihilation operator, a ("hole" creation operator, $a^{\rm h\dagger}$) may in turn be assumed to annihilate a *fermion* or an *antifermion* (create an *antifermion* or a *fermion*) with no possibility for the two assumptions to interfere. Such an "ambivalence" can be made more explicit by setting

$$a(\mathbf{p},\sigma) = a(\mathbf{p},\sigma;|m|), \qquad a^{\rm h\dagger}(\mathbf{p},\sigma) = a^{\rm h\dagger}(\mathbf{p},\sigma;|m|) \tag{14}$$

for both $\psi_f(x)$ and $\psi_{\bar{f}}(x)$. Of course, $\psi_{\bar{f}}(x)$ has nothing to do with the charge-conjugate field that can (noncovariantly) be obtained from $\psi_f(x)$ by applying the usual operation of "particle" $\rightleftharpoons$ "hole" conjugation (i.e., $a \rightleftharpoons a^{\rm h}$, $a^{\rm h\dagger} \rightleftharpoons a^\dagger$): this latter charge-conjugate field type is independently definable both for $\psi_f(x)$ and $\psi_{\bar{f}}(x)$ itself, and can be encountered just *within* either single picture above, as a result of normal ordering. Also, note that due to the *covariant* eigenvalues m and $-m$ marking the pictures in question, an *unambiguous* distinction between "particle" and "hole" now follows the choice of either picture: for instance, if the "fermion" Dirac picture (marked by m) is adopted, then the "hole" label is automatically left assigned to the antifermion, and no ambiguity can arise even when normal ordering is applied. Allowing for (13), one may compactly write

$$\Psi^{(C_{\rm cov})}(x) = \gamma^5 \Psi(x), \quad \bar{\Psi}^{(C_{\rm cov})}(x) = -\bar{\Psi}(x)\gamma^5. \tag{15}$$

These equations strictly define the role now played by γ^5 *as the γ-matrix representing* $C_{\rm cov}$. Note, on the other hand, that from the requirement of invariance of (9) under space inversion $x^\mu \rightarrow x_\mu$, one may still infer the P mirror counterparts of Ψ and $\bar{\Psi}$ as those fields looking (apart from phase factors) like

$$\Psi^{(P)}(x_\mu) = \gamma^0 \Psi(x^\mu), \quad \bar{\Psi}^{(P)}(x_\mu) = \bar{\Psi}(x^\mu)\gamma^0. \tag{16}$$

To see the real advantages of this (apparently redundant) formalism, a better insight into the properties of the internal space $S_{\rm in}$ is needed. Consider the new $S_{\rm in}$ basis obtained from the "Dirac" one $\big(|\,f\rangle, |\,\bar{f}\,\rangle\big)$ by carrying out the rotation

$$|\,f\rangle = \frac{1}{\sqrt{2}}(\,|f^{\rm ch}\rangle + |\bar{f}^{\rm ch}\rangle\,), \quad |\,\bar{f}\,\rangle = \frac{1}{\sqrt{2}}(-|f^{\rm ch}\rangle + |\bar{f}^{\rm ch}\rangle\,). \tag{17}$$

The peculiar feature of such a basis is that $C_{\rm cov}$ is made *diagonal* in it:

$$C_{\rm cov}|f^{\rm ch}\rangle = -|f^{\rm ch}\rangle, \quad C_{\rm cov}|\bar{f}^{\rm ch}\rangle = |\bar{f}^{\rm ch}\rangle. \tag{18}$$

In view of (15), the $C_{\rm cov}$ eigenvalues may just be said to afford the "chiralities" of the associated (either positive- or negative-energy) Fock states covariantly labelled by $|f^{\rm ch}\rangle$ and $|\bar{f}^{\rm ch}\rangle$. A similar (unitary and Hermitian) operator, say, $P_{\rm in}$, can clearly be introduced in $S_{\rm in}$, which, vice versa, is diagonal in the basis $(|\,f\rangle, |\,\bar{f}\,\rangle)$ and has the property of interchanging $|f^{\rm ch}\rangle$ and $|\bar{f}^{\rm ch}\rangle$:

$$P_{\rm in}|f^{\rm ch}\rangle = |\bar{f}^{\rm ch}\rangle, \quad P_{\rm in}|\bar{f}^{\rm ch}\rangle = |f^{\rm ch}\rangle \qquad \big(P_{\rm in}^{-1} = P_{\rm in}^\dagger = P_{\rm in}\big). \tag{19}$$

Since

$$P_{\rm in}|\,f\rangle = |\,f\rangle, \quad P_{\rm in}|\,\bar{f}\,\rangle = -|\,\bar{f}\,\rangle, \tag{20}$$

it is appropriate to interpret $P_{\rm in}$ (apart from a phase constant $\eta = \pm 1$) as an "internal parity" covariant operator. As far as the only positive-energy spectrum is concerned, the $P_{\rm in}$ eigenvalues drawn from (20) may well be assumed to reproduce the *intrinsic parities* (i.e., the zero-momentum P eigenvalues) of the Dirac fermion and antifermion. Such a coincidence can no longer be pursued when also the negative-energy spectrum is included, since the intrinsic parity of a spin-$\frac{1}{2}$ particle (unlike the "internal parity" of it) is not a strict *covariant* eigenvalue and changes sign on passing to negative energies. The fact is that parity P is now to be taken as an operator defined in the whole Fock space (2), with an "external" representation, $P_{\rm ex}$, properly acting on $\mathcal{F}^\circ$ vectors, and an "internal" one, $P_{\rm in}$, properly acting on $S_{\rm in}$ vectors: one should write, e.g.,

$$\Psi^{(P)}(x_\mu) = P_{\rm ex}^\dagger \psi_f(x_\mu) P_{\rm ex}\,\langle f|P_{\rm in} \,+\, P_{\rm ex}^\dagger \psi_{\bar{f}}(x_\mu) P_{\rm ex}\,\langle\,\bar{f}|P_{\rm in}, \tag{21}$$

where a comparison with Eqs. (16) and (20) shows (in full accordance with the anticommutation relation $\gamma^0\gamma^5 + \gamma^5\gamma^0 = 0$) that

$$P_{\rm ex}^{\dagger}\psi_f(x_\mu)P_{\rm ex} = \gamma^0\psi_f(x^\mu), \quad P_{\rm ex}^{\dagger}\psi_{\bar f}(x_\mu)P_{\rm ex} = -\gamma^0\psi_{\bar f}(x^\mu). \tag{22}$$

On passing to the new basis $(|f^{\rm ch}\rangle, |\bar f^{\rm ch}\rangle)$, which may be called the "chiral" basis in $S_{\rm in}$, the field $\Psi(x)$ and its adjoint will read

$$\Psi(x) = \chi_f(x)\langle f^{\rm ch}| + \chi_{\bar f}(x)\langle \bar f^{\rm ch}|, \quad \bar\Psi(x) = |f^{\rm ch}\rangle\,\bar\chi_f(x) + |\bar f^{\rm ch}\rangle\,\bar\chi_{\bar f}$$

$(\bar\chi = \chi^{\dagger}\gamma^0)$ with (23)

$$\chi_f(x) \equiv \frac{1}{\sqrt 2}(1-\gamma^5)\psi_f(x), \quad \chi_{\bar f}(x) \equiv \frac{1}{\sqrt 2}(1+\gamma^5)\psi_{\bar f}(x) \tag{24}$$

and

$$\psi_f = \frac{1}{\sqrt 2}(\chi_f + \chi_{\bar f}), \quad \psi_{\bar f} = \frac{1}{\sqrt 2}(-\chi_f + \chi_{\bar f}), \tag{25}$$

and with (25) being identically valid also for $\bar\psi_f, \bar\psi_{\bar f}$ and $\bar\chi_f, \bar\chi_{\bar f}$. So, in the enlarged framework provided by (2), two (massive) "chiral fields," χ_f and $\chi_{\bar f}$, can spontaneously be introduced, just having *opposite* chiralities and being *on the same footing* as the two "covariant charge-conjugated" Dirac fields ψ_f and $\psi_{\bar f}$. They may themselves be said to be "covariantly conjugated" to each other, but with $P_{\rm in}$ taking the place of $C_{\rm cov}$: if (19) is taken into account, then, by an inspection of (23), it is immediate to see that applying $P_{\rm in}$ to both Ψ and $\bar\Psi$ can equivalently be accomplished by prescribing

$$P_{\rm in} : \chi_f(x) \rightleftharpoons \chi_{\bar f}(x), \quad \bar\chi_f(x) \rightleftharpoons \bar\chi_{\bar f}(x). \tag{26}$$

It appears evident, on the other hand, that $C_{\rm cov}$ is conversely acting *as if*

$$C_{\rm cov} : \begin{cases} \chi_f(x) \to -\chi_f(x),\ \chi_{\bar f}(x) \to \chi_{\bar f}(x) \\ \bar\chi_f(x) \to -\bar\chi_f(x),\ \bar\chi_{\bar f}(x) \to \bar\chi_{\bar f}(x). \end{cases} \tag{27}$$

All this *a fortiori* makes sense in the zero-mass limit, which clearly reduces both (fermion and antifermion) Dirac fields ψ_f and $\psi_{\bar f}$, as given by (25), to simple mixtures of a pure *left-handed* fermion and a pure *right-handed* antifermion (chiral) field (Barut and Ziino, 1993) (a detailed analysis of such spontaneous *two-component* field models for massless spin-$\frac{1}{2}$ particles and antiparticles will be made in Section 4). The general meaning of (26) can be gathered with the help of (13): if we take, e.g., the *intrinsic* mirror counterparts of χ_f and $\chi_{\bar f}$, defined as their respective chirality-conjugate versions

$$\xi_f(x) \equiv \frac{1}{\sqrt 2}(1+\gamma^5)\psi_f(x), \quad \xi_{\bar f}(x) \equiv \frac{1}{\sqrt 2}(1-\gamma^5)\psi_{\bar f}(x), \tag{28}$$

we see that

$$\xi_f(x) = \chi_{\bar{f}}(x), \quad \xi_{\bar{f}}(x) = -\chi_f(x). \tag{29}$$

Thus, no further independent pair of chiral fields can be obtained by chirality inversion; and, in view of (29), it may be argued that χ_f ($\bar{\chi}_f$) and $\chi_{\bar{f}}$ ($\bar{\chi}_{\bar{f}}$) are *themselves* merely acting as the intrinsic mirror counterparts of each other.

By virtue of (25), the "maximally P-violating" *Dirac-field* $V - A$ *current* apparently entering into both lepton and quark weak phenomenologies (Halzen and Martin, 1984) can now find a *natural* theoretical room in the straightforward form of a *chiral-field V current*: using the subscripts a, b to specify the point-fermion types involved in the current, one obtains

$$\bar{\psi}_{\rm b}\gamma^\mu(1-\gamma^5)\psi_{\rm a} \equiv \bar{\chi}_{\rm b}\gamma^\mu\chi_{\rm a}. \tag{30}$$

A similar conclusion could be drawn also for an equivalent $V + A$ current in terms of the respective "covariant charge-conjugate" Dirac antifermion fields $\psi_{\bar{\rm a}} = \gamma^5\psi_{\rm a}$ and $\bar{\psi}_{\bar{\rm b}} = -\gamma^5\bar{\psi}_{\rm b}$:

$$\bar{\psi}_{\bar{\rm b}}\gamma^\mu(1+\gamma^5)\psi_{\bar{\rm a}} \equiv \bar{\chi}_{\bar{\rm b}}\gamma^\mu\chi_{\bar{\rm a}}. \tag{31}$$

So, a pure weakly-interacting point fermion may strictly be referred to as a "chiral" fermion, on which, in view of (26), $P_{\rm in}$ itself will play a "covariant charge-conjugation" role quite similar to the one played by $C_{\rm cov}$ on a "Dirac" fermion: it will give rise to a new, equally allowable chiral-particle description (covariantly conjugated to the starting one) where the original associations "particle" = fermion and "hole" = antifermion appear to be interchanged. This seems even to lead to a *recovery* of P symmetry, as follows from the fact that either in the new covariant form (30) or (31) the parity matrix γ^0 is to be applied *directly* to χ and $\bar{\chi}$, rather than (as usual) to ψ and $\bar{\psi}$:

$$P : \chi(x^\mu) \to \gamma^0\chi(x_\mu), \quad \bar{\chi}(x^\mu) \to \bar{\chi}(x_\mu)\gamma^0. \tag{32}$$

More precisely, setting – as prescribed by (2) – $P = P_{\rm in}P_{\rm ex}$ ($= P_{\rm ex}P_{\rm in}$) ($P_{\rm ex}$ standing for the "external" parity, properly defined in $\mathcal{F}^\circ$, and $P_{\rm in}$ for the "internal" parity, properly defined in $S_{\rm in}$) one can see that the peculiar left–right spatial asymmetry shown by the pure weak couplings is here accounted for as a mere (maximal) $P_{\rm ex}$ violation, which, suitably combined with a (maximal) $P_{\rm in}$ violation, does not prevent P itself from being left, on the whole, a symmetry operation. A simple comparison of (26) with (32), via Eqs. (29) and (28), reveals that *applying* $P_{\rm ex}$ *to a chiral-field V current is quite the same as usually applying* P *to the corresponding Dirac-field* $V - A$ *current*. If by $P_{\rm st}$ we denote an operator just reproducing the "standard" formal way of applying P according to the $V - A$ scheme, we may then put $P_{\rm ex} = P_{\rm st}$ and $P = P_{\rm in}P_{\rm st}$. Of course, as long as only a current of the (Dirac) type $\bar{\psi}\gamma^\mu\psi$ is taken into account, the effect of P strictly coincides with the effect of $P_{\rm st}$, so that no distinction can yet be made between P

and $P_{\rm st}$: a glance at both (26) and (25) points out that the $P_{\rm in}$ behavior is irrelevant to such a current. Quite different is the case of the chiral-field currents (30) and (31) taken alone, since they, on the contrary, are turned into *each other* as an effect of $P_{\rm in}$: thus also $P_{\rm in}$ (and not only $P_{\rm ex}$) is maximally violated by them, the overall result being that they may still be singly P-invariant (despite their $P_{\rm ex}$-violating character). This ultimately means that *applying what is only an "external" parity operation* (as it is usually done) *would not exhaust the real effects produced by space reflection on a pure weakly-interacting fermion system*: some "internal" nontrivial effects (really able to restore ordinary mirror symmetry) would be indeed neglected, which should involve a yet unexplored, complementary aspect of the intrinsic nature itself of spin-$\frac{1}{2}$ point fermions. The physical contents of these further effects of space inversion will be made clear in the next section, where just a *dual* (either "Dirac" or "chiral") massive fermion model, generally based on the coexistence of two *anticommuting* (scalar and pseudoscalar) charge varieties, is outlined.

Such a restitution of P symmetry to the "maximally P-violating" phenomenology is consistently supplemented by a parallel restitution of C symmetry, C being the ordinary (noncovariant) charge conjugation. The key-novelty is afforded again by (25), which similarly requires, either for the chiral-field current (30) or (31), a *direct* application of C to χ and $\bar{\chi}$. Using the symbol C also to denote the associated 4×4 unitary matrix, we have, e.g., that applying C to (30) now means making the substitutions

$$\chi_{\rm a} \to \chi_{\rm a}^{(C)} = C\widetilde{\chi}_{\rm a}^{\dagger}, \quad \bar{\chi}_{\rm b} \to \bar{\chi}_{\rm b}^{(C)} = \widetilde{\chi}_{\rm b}C^{\dagger}\gamma^{0} \tag{33}$$

where $\widetilde{\chi}$ stands for the transpose of χ and C is, as usual, such that

$$C\,\widetilde{\gamma}^{\mu\dagger}C^{\dagger} = -\gamma^{\mu}, \quad C\,\widetilde{\gamma}^{5\dagger}C^{\dagger} = -\gamma^{5}. \tag{34}$$

In this way C will also induce *chirality inversion* besides ordinarily acting on Dirac fields; which clearly ensures C symmetry to be restored (provided that normal ordering is applied): as an effect of (33), it will turn out that the "charge conjugate" of (30) is indeed a $V + A$ (rather than a $V - A$) "hole" current. Since (33) generally implies

$$\psi_{\bar{f}}^{(C)}(x) = -\gamma^{5}\psi_{f}^{(C)}(x), \quad \bar{\psi}_{\bar{f}}^{(C)}(x) = \bar{\psi}_{f}^{(C)}(x)\gamma^{5}, \tag{35}$$

it is immediate to see, by a comparison with (13), that the C operator may now be written down as $C = P_{\rm in}C_{\rm st}\,(= C_{\rm st}P_{\rm in})$ with $C_{\rm st}$ exactly reproducing the "standard" formal way of applying C according to the $V - A$ scheme. This is just how C is to be represented in the "dressed" Fock space (2). Of course, no real distinction can yet emerge between C and $C_{\rm st}$, as far as their effects on a current like $\bar{\psi}\gamma^{\mu}\psi$ are concerned: the reason is because by writing $\psi \to C\widetilde{\bar{\psi}}^{\dagger}$ one may indifferently mean not only

$$\frac{1}{2}(1 \mp \gamma^5)\psi \longrightarrow \frac{1}{2}(1 \mp \gamma^5)C\widetilde{\psi}^\dagger$$

(with $C = C_{st}$ as in the usual formalism) but also

$$\frac{1}{2}(1 \mp \gamma^5)\psi \longrightarrow \frac{1}{2}C[(1 \mp \widetilde{\gamma}^{5\dagger})\widetilde{\psi}^\dagger] = \frac{1}{2}(1 \pm \gamma^5)C\widetilde{\psi}^\dagger$$

(with $C = P_{in}C_{st}$ as in the new formalism). The C_{st} and $P_{in}C_{st}$ effects are made fully distinguishable, on the contrary, just when a single chiral-field current like (30) or (31) is involved: C_{st} will act in such a way as to be (maximally) violated, while $P_{in}C_{st}$ in such a way as to be left a symmetry operation.

Hence, in the enlarged framework here considered, it is only when dealing with a Dirac-field V current that we may really put $P = P_{st}$ and $C = C_{st}$. Note, nevertheless, that in this same framework it always results

$$CP = C_{st}P_{st} = (CP)_{st} \tag{36}$$

whether a Dirac- or chiral-field V current is individually involved. If it is further considered that "chiral" fermions would also be manifest C_{cov} eigenstates which are taken by P_{in} into their antifermion counterparts, it may therefore be guessed that the recovered P mirror symmetry for "chiral" fermions should essentially amount to CP symmetry itself, with C thus really acting on them just like an *identity*. A direct confirmation can be obtained by singly applying C $(= P_{in}C_{st})$ and P $(= P_{in}P_{st})$ to a "dressed" Fock state of the type $|1_{\mathbf{p},\sigma}\rangle|f^{ch}\rangle$, with $|1_{\mathbf{p},\sigma}\rangle$ denoting an usual occupied "particle" (= fermion) state of momentum $\mathbf{p}$ and helicity σ in $\mathcal{F}^\circ$. The fact is that

$$C\,|1_{\mathbf{p},\sigma}\rangle|f^{ch}\rangle = \left|1^{(h)}_{\mathbf{p},\sigma}\right\rangle\left|\bar{f}^{ch}\right\rangle, \qquad P\,|1_{\mathbf{p},\sigma}\rangle|f^{ch}\rangle = |1_{-\mathbf{p},-\sigma}\rangle|\bar{f}^{ch}\rangle, \tag{37}$$

where both transformed "dressed" Fock states are also belonging to a *new* picture (marked by $|\bar{f}^{ch}\rangle$ and covariantly conjugated to the starting one) in which by "particle" the corresponding "chiral" antifermion is meant: so, the occupied "hole" state $|1^{(h)}_{\mathbf{p},\sigma}\rangle|\bar{f}^{ch}\rangle$ is nothing but the original *fermion* state as re-expressed in such a picture, while the occupied (mirror) "particle" state $|1_{-\mathbf{p},-\sigma}\rangle|\bar{f}^{ch}\rangle$ is rather a "chiral" *antifermion* (and no longer a "chiral" *fermion*) state. As will be shown in the next section, these can more straightforwardly be seen to correspond to a $C = 1$ and a $P = CP = (CP)_{st}$ effect, if an explicit use is made of a symmetrized "particle–hole" formalism. All that, of course, makes sense, provided that a "Dirac" and a "chiral" fermion are supposed to embody two mere *complementary* and *mutually exclusive* internal attitudes of one and the same spin-$\frac{1}{2}$ point particle. But which would be the reasons for the coexistence of two such (apparently incompatible) fermion natures? Answering this question may indeed be decisive for tracing the origin itself of what is known as the "maximal parity-violation" effect.

3. A DUAL – EITHER "DIRAC" OR "CHIRAL" – MODEL OF A MASSIVE SPIN-$\frac{1}{2}$ POINT PARTICLE

An insight into the above results can be gained by introducing two general one-particle "charge" operators, Q and Q^{ch}, the former being diagonal in the "Dirac" S_{in}-basis $\big(|\,f\rangle, |\,\bar{f}\,\rangle\big)$ (with eigenvalues $\pm q$) and the latter in the "chiral" S_{in}-basis $\big(|\,f^{\mathrm{ch}}\rangle, |\,\bar{f}^{\,\mathrm{ch}}\rangle\big)$ (with eigenvalues $\pm q^{\mathrm{ch}}$). In view of (3), (18), (19), and (20), one has

$$C_{\mathrm{cov}}Q = -QC_{\mathrm{cov}}, \quad P_{\mathrm{in}}Q = QP_{\mathrm{in}} \tag{38}$$

$$P_{\mathrm{in}}Q^{\mathrm{ch}} = -Q^{\mathrm{ch}}P_{\mathrm{in}}, \quad C_{\mathrm{cov}}Q^{\mathrm{ch}} = Q^{\mathrm{ch}}C_{\mathrm{cov}}. \tag{39}$$

Hence, Q is a *scalar* quantity anticommuting with C_{cov}, while Q^{ch} is a *pseudoscalar* quantity anticommuting with P_{in}; so that C_{cov} and P_{in} may strictly be said to stand for a *scalar*- and a *pseudoscalar-charge* conjugation operator, respectively. It may be asserted, moreover, that the "Dirac" internal states $(|\,f\rangle, |\,\bar{f}\,\rangle)$ should typically behave like a *net* pair of scalar-charge conjugated eigenstates—see Eqs. (3)—while the "chiral" ones $(|\,f^{\mathrm{ch}}\rangle, |\,\bar{f}^{\,\mathrm{ch}}\rangle)$ like a *net* pair of pseudoscalar-charge conjugated eigenstates—see Eqs. (19). This is to be related to the fact Q and Q^{ch} are themselves two *anticommuting* operators,

$$QQ^{\mathrm{ch}} + Q^{\mathrm{ch}}Q = 0, \tag{40}$$

whose squares clearly satisfy the commutation relations

$$\left[Q^2, Q^{\mathrm{ch}}\right] = \left[Q^{\mathrm{ch}2}, Q\right] = 0. \tag{41}$$

Either Q or Q^{ch}, if applied (from the right) to the fermion–antifermion field $\Psi(x)$, is automatically able to *superselect* that internal representation of $\Psi(x)$—either (8) or (23)—which may diagonalize it. Hence it can be argued that the same massive spin-$\frac{1}{2}$ point fermion and related antifermion may both display, in principle, a *dual* intrinsic nature as an alternate pair of *superselected* Q or Q^{ch} eigenstates (Barut, 1972, 1973). If so, then, e.g., the "true" operation of fermion $\to$ antifermion covariant conjugation should strictly be identified with $C_{\mathrm{cov}}P_{\mathrm{in}}$, although one has that $C_{\mathrm{cov}}P_{\mathrm{in}}$ is just reducible to C_{cov} when acting on $|\,f\rangle$ and to P_{in} when acting on $|\,f^{\mathrm{ch}}\rangle$:

$$C_{\mathrm{cov}}P_{\mathrm{in}}|\,f\rangle = C_{\mathrm{cov}}|\,f\rangle, \quad C_{\mathrm{cov}}P_{\mathrm{in}}|\,f^{\mathrm{ch}}\rangle = P_{\mathrm{in}}|\,f^{\mathrm{ch}}\rangle \tag{42}$$

($C_{\mathrm{cov}}P_{\mathrm{in}} = -P_{\mathrm{in}}C_{\mathrm{cov}}$). In the former case the fermion would behave *as if* it were a pure scalar-charge (i.e., "Dirac") particle, while in the latter *as if* it were a pure pseudoscalar-charge (i.e., "chiral") particle. Yet it would be in *either* (and not only in the former) case that P and C_{cov} symmetries can be singly respected (Barut, 1982), the difference being merely that P and C_{cov} would result to play *interchanged* internal roles on passing from one to the other case. Actually, Eqs. (42)

show that a "chiral" fermion as compared with a "Dirac" one would conversely stand for a $C_{\rm cov}$ (and not an intrinsic P) eigenstate which instead is turned by P into the corresponding "chiral" antifermion: in such a case, therefore, P itself (thanks to $P_{\rm in}$) would take the place of $C_{\rm cov}$ as a "covariant charge-conjugation" operation.

On the other hand, coming back to the "dressed" representation, $C = P_{\rm in}C_{\rm st}$, of the ordinary (noncovariant) charge conjugation C in the new Fock space (2), we clearly have that the "bare" contribution $C_{\rm st}$ will take "particle" ("hole") into "hole" ("particle") no matter whether a "Dirac" or "chiral" fermion (antifermion) is being dealt with. So, according to either case considered, $C_{\rm st}$ may in turn be said to stand for the noncovariant analogue of the "covariant charge-conjugation" operations $C_{\rm cov}$ and $P_{\rm in}$, respectively. What is really peculiar, however, to $C_{\rm st}$ (as well as to $CP = C_{\rm st}P_{\rm st}$) is the noncovariant character itself, which allows it to act in the strict framework of either *single* picture with "particle" = fermion (and "hole" = antifermion) or "particle" = antifermion (and "hole" = fermion). This cannot happen for $C_{\rm cov}$ (in the "Dirac" case) and for $P_{\rm in}$ (in the "chiral" case), since a "covariant charge-conjugation" implies just a change from the former picture (properly in terms of positive- and negative-energy *fermions*) to the latter (properly in terms of positive- and negative-energy *antifermions*) or vice versa. Such a difference, if seen from a reversed viewpoint, can fully explain how the P $(= P_{\rm in}P_{\rm st})$ mirror image of a "chiral" fermion may already amount to a CP mirror image of it: due to the $P_{\rm in}$ contribution, *applying P to a "chiral" fermion will also signify going over to a new "particle–hole" picture where one has "particle" = antifermion (rather than "particle" = fermion) and where a space-inverted "particle" state correspondingly stands for an antifermion (rather than still for a fermion) state*. The physical equivalence thus resulting between the recovered P mirror symmetry and the well-known CP mirror symmetry can be given a more straightforward (though only *effective*) representation by use of a symmetrized "particle–hole" formalism (as obtained via normal ordering). A formalism like this is no longer strictly covariant but enables one to evaluate the C and P individual effects even without passing to a new "particle–hole" picture. To see it, consider also the "hole" version of transformation (25),

$$\psi_f^{(\rm h)} = \frac{1}{\sqrt{2}}\big(\chi_f^{(\rm h)} + \chi_{\bar f}^{(\rm h)}\big), \quad \psi_{\bar f}^{(\rm h)} = \frac{1}{\sqrt{2}}\big(-\chi_f^{(\rm h)} + \chi_{\bar f}^{(\rm h)}\big), \tag{43}$$

where the fields $\psi_f^{(\rm h)}, \psi_{\bar f}^{(\rm h)}$ and $\chi_f^{(\rm h)}, \chi_{\bar f}^{(\rm h)}$ are obtained from the corresponding fields $\psi_f, \psi_{\bar f}$ and $\chi_f, \chi_{\bar f}$ by simply making (in their Fourier expansions) the substitutions $a \to a^{\rm h}$, $a^{\rm h\dagger} \to a^{\dagger}$. Take then, e.g., the normally ordered "particle–hole" chiral-field picture with "particle" = fermion (and "hole" = antifermion): there will symmetrically enter both the (negative chirality) "particle" field χ_f and the (positive chirality) "hole" field $\chi_{\bar f}^{(\rm h)}$ (along with their adjoints $\bar\chi_f$ and $\bar\chi_{\bar f}^{(\rm h)}$). It follows that the operation

$$P_{\rm in} : \chi_f\,(\bar\chi_f) \longrightarrow \chi_{\bar f}\,(\bar\chi_{\bar f}), \quad \chi_{\bar f}^{\rm (h)}\left(\bar\chi_{\bar f}^{\rm (h)}\right) \longrightarrow \chi_f^{\rm (h)}\left(\bar\chi_f^{\rm (h)}\right) \tag{44}$$

cannot globally be distinguished from

$$C_{\rm st} : \chi_f\,(\bar\chi_f) \longrightarrow \chi_f^{\rm (h)}\left(\bar\chi_f^{\rm (h)}\right), \quad \chi_{\bar f}^{\rm (h)}\left(\bar\chi_{\bar f}^{\rm (h)}\right) \longrightarrow \chi_{\bar f}\,(\bar\chi_{\bar f}), \tag{45}$$

so that $P_{\rm in}$ itself may equivalently be ascribed an *effective* behavior like $P_{\rm in} = C_{\rm st}$ (as if it were directly defined in the "bare" Fock space $\mathcal{F}^\circ$). Analogous, *effective* behaviors on $\mathcal{F}^\circ$ states may be thought of, accordingly, for C $(= P_{\rm in}C_{\rm st})$ and P $(= P_{\rm in}P_{\rm st})$, which just amount to $C = 1$ and $P = CP = C_{\rm st}P_{\rm st}$. So, after all, a "chiral" particle can soundly be said to be a particle *looking C-invariant*, and this should indeed be taken as a noncovariant analogue to its apparent being a strict $C_{\rm cov}$ (i.e., scalar-charge conjugation) eigenstate.

With the help of both (1) and (20), Eq. (9) can be recast into the more convenient form

$$i\gamma^\mu\partial_\mu\Psi(x) = |m|\,\Psi^{(P_{\rm in})}(x) \tag{46}$$

where

$$\Psi^{(P_{\rm in})}(x) \equiv \Psi(x)\,P_{\rm in} = \psi_f(x)\,\langle f| - \psi_{\bar f}(x)\,\langle \bar f|. \tag{47}$$

A field equation like (46) is actually derivable from the Hermitian free Lagrangian

$$\mathcal{L}\big(\Psi, \Psi^{(P_{\rm in})}, \bar\Psi, \bar\Psi^{(P_{\rm in})}, \ldots; |m|\big) = \frac{1}{4}\Big[i\left(\bar\Psi\gamma^\mu\partial_\mu\Psi + \bar\Psi^{(P_{\rm in})}\gamma^\mu\partial_\mu\Psi^{(P_{\rm in})}\right) + \text{H.c.}\Big]$$
$$-\frac{1}{2}|m|\left(\bar\Psi\Psi^{(P_{\rm in})} + \bar\Psi^{(P_{\rm in})}\Psi\right) \tag{48}$$

where $\bar\Psi^{(P_{\rm in})} = P_{\rm in}\bar\Psi$. In (48), one has that Ψ, $\Psi^{(P_{\rm in})}$, $\bar\Psi$ and $\bar\Psi^{(P_{\rm in})}$ are *single* field variables; by this it is understood that χ_f and $\chi_{\bar f}$ are subject *from the beginning* to the "chiral condition" (24) (automatically fixing the link between ψ_f and $\psi_{\bar f}$). A glance at (48) shows that $\mathcal{L}$ is not only manifestly $P_{\rm in}$-invariant, but also P-invariant, as can be immediately checked by applying the usual effective prescription

$$P : \partial_\mu \rightarrow \partial^\mu, \quad \gamma^\mu \rightarrow \gamma^0\gamma^\mu\gamma^0. \tag{49}$$

This (fully covariant) outcome is *independent* of the special $S_{\rm in}$ representation chosen for the fields Ψ, $\Psi^{(P_{\rm in})}$, $\bar\Psi$ and $\bar\Psi^{(P_{\rm in})}$; so one has that parity invariance consistently holds *even when the chiral $S_{\rm in}$ representation is adopted*. Quite a similar remark applies to the (substantial) invariance of $\mathcal{L}$ under the ordinary operation of charge conjugation,

$$\Psi \rightarrow C\widetilde\Psi^\dagger, \quad \Psi^\dagger \rightarrow \widetilde\Psi C^\dagger; \quad \Psi^{(P_{\rm in})} \rightarrow C\widetilde\Psi^{(P_{\rm in})\dagger}, \quad \Psi^{(P_{\rm in})\dagger} \rightarrow \widetilde\Psi^{(P_{\rm in})}C^\dagger, \tag{50}$$

with the matrix C fulfilling the usual conditions (34).

Global phase invariance of the Lagrangian (48) yields a manifestly $P_{\rm in}$-invariant, conserved free current like

$$J \equiv J^{\mu} = \frac{1}{2}\big[\,\bar{\Psi}\gamma^{\mu}\Psi \,+\, \bar{\Psi}^{(P_{\rm in})}\gamma^{\mu}\Psi^{(P_{\rm in})}\big]. \tag{51}$$

By use of the closure relation $|\,f\,\rangle\langle\,f\,| \,+\, |\,\bar{f}\,\rangle\langle\,\bar{f}\,| = 1$ (throughout this paper the identity operator in $S_{\rm in}$ is simply denoted by 1) such a current can essentially be reduced to the "bare" form

$$J^{\mu} = \bar{\psi}_f\gamma^{\mu}\psi_f = \bar{\psi}_{\bar{f}}\,\gamma^{\mu}\psi_{\bar{f}} = \frac{1}{2}\big[\,\bar{\chi}_f\gamma^{\mu}\chi_f \,+\, \bar{\chi}_{\bar{f}}\,\gamma^{\mu}\chi_{\bar{f}}\,\big] \tag{52}$$

acting in the strict Fock space $\mathcal{F}^{\circ}$. Likewise one has $J = J^{(C_{\rm cov})}$, and this corresponds to the fact that (51) is chirality-invariant. The form (52) can be suitably "dressed" to give the two distinct, *scalar*- and *pseudoscalar*-charge, conserved free currents

$$\mathcal{J}^{(Q)} = Q\,J = J\,Q, \quad \mathcal{J}^{(Q^{\rm ch})} = Q^{\rm ch}J = J\,Q^{\rm ch}, \tag{53}$$

which act in the whole space (2) and are, according to (38) and (39), such that

$$C_{\rm cov}\mathcal{J}^{(Q)} = -\mathcal{J}^{(Q)}C_{\rm cov}, \qquad P_{\rm in}\mathcal{J}^{(Q)} = \mathcal{J}^{(Q)}P_{\rm in} \tag{54}$$

$$P_{\rm in}\mathcal{J}^{(Q^{\rm ch})} = -\mathcal{J}^{(Q^{\rm ch})}P_{\rm in}, \qquad C_{\rm cov}\;\mathcal{J}^{(Q^{\rm ch})} = \mathcal{J}^{(Q^{\rm ch})}C_{\rm cov}. \tag{55}$$

These can also (more properly) be inferred by exploiting the invariance of (48) under two individual kinds of global gauge transformations applying to $S_{\rm in}$ vectors, such that

$$\Psi \to \Psi e^{i\,Q\alpha},\ \bar{\Psi} \to e^{-i\,Q\alpha}\bar{\Psi}\,;\ \Psi^{(P_{\rm in})} \to \Psi^{(P_{\rm in})}e^{i\,Q\alpha},\ \bar{\Psi}^{(P_{\rm in})} \to e^{-i\,Q\alpha}\,\bar{\Psi}^{(P_{\rm in})} \tag{56}$$

and

$$\Psi \to \Psi e^{i\,Q^{\rm ch}\beta}, \bar{\Psi} \to e^{-i\,Q^{\rm ch}\beta}\bar{\Psi}\,;\ \Psi^{(P_{\rm in})} \to \Psi^{(P_{\rm in})}e^{i\,Q^{\rm ch}\beta}, \bar{\Psi}^{(P_{\rm in})} \to e^{-i\,Q^{\rm ch}\beta}\bar{\Psi}^{(P_{\rm in})}, \tag{57}$$

respectively (α and β being two constant real angles). In particular, note that the invariance with respect to the substitutions (57) may hold also for the mass sector of (48) by virtue of the chiral condition (24). The *vector* and *axial-vector* behaviors of $\mathcal{J}^{(Q)}$ and $\mathcal{J}^{(Q^{\rm ch})}$ can be explicitly checked as follows:

$$\begin{cases} P^{\dagger}\mathcal{J}^{(Q)\mu}(x_{\nu})P = (P_{\rm in}^{\dagger}Q\,P_{\rm in})[P_{\rm ex}^{\dagger}J^{\mu}(x_{\nu})P_{\rm ex}] = \mathcal{J}_{\mu}^{(Q)}(x^{\nu}) \\ P^{\dagger}\mathcal{J}^{(Q^{\rm ch})\mu}(x_{\nu})P = (P_{\rm in}^{\dagger}Q^{\rm ch}P_{\rm in})[P_{\rm ex}^{\dagger}J^{\mu}(x_{\nu})P_{\rm ex}] = -\mathcal{J}_{\mu}^{(Q^{\rm ch})}(x^{\nu}). \end{cases} \tag{58}$$

The corresponding behaviors of their normally ordered versions under the (ordinary) charge-conjugation C are

$$C^{\dagger}\big(:\mathcal{J}^{(Q)}:\big)\,C = -\big(:\mathcal{J}^{(Q)}:\big)\,, \quad C^{\dagger}\big(:\mathcal{J}^{(Q^{\rm ch})}:\big)\,C = \big(:\mathcal{J}^{(Q^{\rm ch})}:\big), \tag{59}$$

with $C = P_{\rm in}C_{\rm st}$ and

$$C_{\rm st}^{\dagger} Q\, C_{\rm st} = Q, \quad C_{\rm st}^{\dagger} Q^{\rm ch}\, C_{\rm st} = Q^{\rm ch}, \quad C_{\rm st}^{\dagger}(:J:)\, C_{\rm st} = -(:J:). \tag{60}$$

Note, on the other hand, that both $(:\mathcal{J}^{(Q)}:)$ and $(:\mathcal{J}^{(Q^{\rm ch})}:)$ behave *identically* under CP $(= C_{\rm st}P_{\rm st})$.

As for the scalar-charge current $\mathcal{J}^{(Q)}$, the "dressing" one-particle charge operator relevant to it can be expressed as

$$Q = Q(q) = q\, P_{\rm in} = q\left(|\, f\rangle\langle f\,| \,-\, |\,\bar{f}\,\rangle\langle\,\bar{f}\,|\right), \tag{61}$$

q $(= \mp|q|)$ being the given Q eigenvalue associated with $|\, f\rangle$. Substituting (61) splits up $\mathcal{J}^{(Q)}$ into the sum of two currents which are the "covariant charge-conjugates" of each other:

$$\mathcal{J}^{(Q)} = q\,(|\, f\rangle\bar{\psi}_f\, \gamma^{\mu}\psi_f\langle f| - |\,\bar{f}\,\rangle\bar{\psi}_{\bar{f}}\, \gamma^{\mu}\psi_{\bar{f}}\langle\,\bar{f}\,|\,). \tag{62}$$

They are the "Dirac"-fermion current, marked by $|\, f\rangle\langle f\,|$, and the "Dirac"-antifermion one, marked by $|\,\bar{f}\,\rangle\langle\,\bar{f}\,|$, the former being associated with a proper-mass root $+m$ and the latter with a proper-mass root $-m$. Of course, both *alternative* pictures in terms of such individual currents are consistent with QED, since a Dirac bilinear form such as $\bar{\psi}\,\gamma^{\mu}\psi$ is left unvaried by proper-mass reversal $\psi \to \gamma^5\psi\,,\ \bar{\psi} \to -\bar{\psi}\gamma^5$. Under normal ordering, either the fermion or antifermion sector of $\mathcal{J}^{(Q)}$ may just be recast so as to look like a *complete* (antisymmetric) "particle + hole" current (marked by a *single* proper-mass sign): the difference is that in the former case, one is choosing "particle" = fermion (and "hole" = antifermion) while in the latter case, one is conversely choosing "particle" = antifermion (and "hole" = fermion). The general free Lagrangian (48) can be made invariant also under local U(1) transformations generated by Q, provided that a minimal coupling term like $-\mathcal{J}^{(Q)}A$ is inserted into it, with $A \equiv A_{\mu}$ being a (massless) *vector* field, such that

$$P_{\rm in}^{\dagger}\, A_{\mu}(x_{\nu})\, P_{\rm in} = A_{\mu}(x_{\nu})\;, \qquad P_{\rm ex}^{\dagger}\, A_{\mu}(x_{\nu})\, P_{\rm ex} = -A_{\mu}(x^{\nu}). \tag{63}$$

The additional presence of $-\mathcal{J}^{(Q)}A$ clearly breaks the original covariance of (48) under rotations in $S_{\rm in}$. The coupling term in question is *double*-structured as well: it actually merges two *equivalent*, and only *in turn* available, coupling terms, singly involving the fermionic and the antifermionic covariant current embodied in $\mathcal{J}^{(Q)}$. One may still deal with $C_{\rm cov}$ as a symmetry operation (represented by the chirality transformation $\psi \;\to \gamma^5\psi\,,\ \bar{\psi}\;\to -\bar{\psi}\gamma^5$) provided that it is supposed to apply to the whole interacting system (including A_{μ}) so as to yield

$$C_{\rm cov}^{\dagger}\big(\mathcal{J}^{(Q)\mu}A_{\mu}\big)\, C_{\rm cov} = \big(-\,\mathcal{J}^{(Q)\mu}\big)(-A_{\mu}), \tag{64}$$

with $-A_{\mu}$ being the "covariant charge-conjugate" (besides the ordinary charge-conjugate) of A_{μ}. This looks just like a *scalar-charge conjugation* covariant operation (extended to A_{μ}). Note that an equivalent way of defining $C_{\rm cov}$ as a symmetry operation can be obtained by setting, instead of (64),

$$C_{\rm cov}:\ |f\rangle\langle f| \rightleftharpoons |\bar{f}\rangle\langle\bar{f}|,\quad q\rightleftharpoons -q;\quad A_\mu \rightleftharpoons A_\mu. \tag{65}$$

According to (65), $C_{\rm cov}$ stands again (as in the free-particle case) for a genuine *proper-mass reversal* operation, which acts strictly within $S_{\rm in}$ and leaves both Q and A_μ unaffected. Of course, the associated property of γ^5-invariance, if taken alone, holds as well for the single (fermionic and antifermionic) sectors of $-\mathcal{J}^{(Q)}A$; this indeed corresponds to the fact that such sectors are individually invariant under the operation

$$q\longrightarrow -q,\quad m\longrightarrow -m,\quad A_\mu\longrightarrow -A_\mu, \tag{66}$$

which can indifferently be seen as the "bare" analogue of (64) or (65).

Consider now the pseudoscalar-charge current $\mathcal{J}^{(Q^{\rm ch})}$, endowed with a "dressing" charge operator like

$$Q^{\rm ch}=Q^{\rm ch}(q^{\rm ch})=-q^{\rm ch}C_{\rm cov}=q^{\rm ch}\left(|f^{\rm ch}\rangle\langle f^{\rm ch}|\,-\,|\bar{f}^{\rm ch}\rangle\langle\bar{f}^{\rm ch}|\right), \tag{67}$$

$q^{\rm ch}$ $(=\mp|q^{\rm ch}|)$ being the given $Q^{\rm ch}$ eigenvalue associated with $|f^{\rm ch}\rangle$. At first sight, no *individual* chiral-field currents seem to be involved in the fermionic (i.e., $q^{\rm ch}\,|f^{\rm ch}\rangle J\langle f^{\rm ch}|$) and antifermionic (i.e., $-q^{\rm ch}\,|\bar{f}^{\rm ch}\rangle J\langle\bar{f}^{\rm ch}|$) sectors of it. Despite this, the virtual existence of *superselected* roles for such currents may still be recognized if $\mathcal{J}^{(Q^{\rm ch})}$ is suitably rewritten in the form

$$\mathcal{J}^{(Q^{\rm ch})}=\frac{1}{2}q^{\rm ch}\left(\mathcal{J}^{\rm ch}-\mathcal{J}^{{\rm ch}(P_{\rm in})}\right), \tag{68}$$

where

$$\mathcal{J}^{\rm ch}=|f^{\rm ch}\rangle\bar{\chi}_f\,\gamma^\mu\chi_f\langle f^{\rm ch}|-|\bar{f}^{\rm ch}\rangle\bar{\chi}_{\bar{f}}\,\gamma^\mu\chi_{\bar{f}}\langle\bar{f}^{\rm ch}| \tag{69}$$

and $\mathcal{J}^{{\rm ch}(P_{\rm in})}=P_{\rm in}^\dagger\mathcal{J}^{\rm ch}P_{\rm in}$, and where applying $P_{\rm in}$ can now be effectively expressed by exploiting, instead of (26), the generalized prescription

$$P_{\rm in}:\chi_f(x)\ \rightleftharpoons\ \chi_{\bar{f}}(x),\quad \bar{\chi}_f(x)\ \rightleftharpoons\ \bar{\chi}_{\bar{f}}(x);\quad q^{\rm ch}\rightleftharpoons -q^{\rm ch}. \tag{70}$$

The fact is that as long as a mass term is present in the Lagrangian (48), neither $\mathcal{J}^{\rm ch}$ nor $\mathcal{J}^{{\rm ch}(P_{\rm in})}$ may be independent and divergenceless, and only $\mathcal{J}^{(Q^{\rm ch})}$ as a whole may really turn out to be a *conserved* pseudoscalar-charge current (Ziino, 1996). Strictly speaking, and still in agreement with the standard electroweak formulation, one is thus led to conclude that *pure chiral-field-current gauge couplings cannot be conceived for originally massive particles.*

4. SPONTANEOUS TWO-COMPONENT UNIVERSAL MODELS OF A MASSLESS SPIN-$\frac{1}{2}$ PARTICLE AND OF ITS ANTIPARTICLE, BEING THE PURE CHIRALITY-CONJUGATES OF EACH OTHER

The "internal parity" covariant operator defined by either (19) or (20) is such that (by use of the chiral representation in S_{in}) one gets

$$\Psi(x)\,P_{\text{in}} = \chi_f(x)\,\langle \bar{f}^{\text{ch}}| + \chi_{\bar{f}}(x)\,\langle f^{\text{ch}}|, \tag{71}$$

$\chi_f(x)$ and $\chi_{\bar{f}}(x)$ being the (fermion and antifermion) chiral-field solutions given by (24). A comparison of both Eq. (71) and its adjoint with Eqs. (23) clearly leads to (26). In view of (13), it then follows that P_{in} essentially amounts to a *chirality inversion* operator for the two fields χ_f, $\chi_{\bar{f}}$ and their adjoints. Such a property should be expected to take on a special significance in the zero-mass case, as therein a chirality eigenvalue can give also a direct information on the particle *helicity* (according to the sign of the particle energy).

For $|m| \to 0$, the Lagrangian (48) reduces to

$$\mathcal{L}(|m| \to 0) = \frac{1}{4}\big[i\left(\bar{\Psi}\gamma^\mu\partial_\mu\Psi + \bar{\Psi}^{(P_{\text{in}})}\gamma^\mu\partial_\mu\Psi^{(P_{\text{in}})}\right) + \text{H.c.}\big]. \tag{72}$$

Such a form still has manifest covariance extended to S_{in}, and clearly *retains* P-invariance (even when use is made of the chiral basis in S_{in}): the latter property can immediately be shown up by applying the covariant prescription (49). The peculiar feature of (72) is, however, the fact that (due to the absence of mass terms) it can be split into two independent Lagrangians being the P_{in} mirror counterparts of each other. Actually, the above-seen *dual* – either "Dirac" or "chiral" – nature of a massive spin-$\frac{1}{2}$ point fermion is now lost, since a massless (and definite helicity) fermion is bound to be a "definite chirality" particle. An insight into this can be gained by an inspection of (25): as $|m| \to 0$, one gets that the two "scalar-charge conjugated" Dirac fields ψ_f and $\psi_{\bar{f}}$ may only survive as *mixtures* of the two "pseudoscalar-charge conjugated" chiral fields χ_f and $\chi_{\bar{f}}$. The zero-mass limit should therefore lead to a *permanent pseudoscalar-charge superselection rule*, as opposed to a *permanent scalar-charge "anti-superselection" rule*: on the basis of what stated in Secs. 2, 3, a (strictly "chiral") massless spin-$\frac{1}{2}$ particle and its antiparticle can exist only as a pair of *pseudoscalar*-charge conjugated eigenstates, which are correspondingly bound to show *vanishing* scalar-charge expectation values. For $m = 0$, one also has that the two fields χ_f, $\chi_{\bar{f}}$ themselves can be made mutually distinguishable in that they annihilate (create) *opposite-helicity* "particle" ("hole") states: more precisely, the χ_f and $\chi_{\bar{f}}$ Fourier expansions will contain "particle" annihilation and "hole" creation operators of the only types $a(\mathbf{p}, \sigma_-)$, $a^{\text{h}\dagger}(\mathbf{p}, \sigma_+)$ and $a(\mathbf{p}, \sigma_+)$, $a^{\text{h}\dagger}(\mathbf{p}, \sigma_-)$, respectively, $\sigma_\mp$ denoting the (negative and positive) helicity eigenvalues. So, after all, taking an explicit account of the chiral condition (24), one may appropriately reduce the Lagrangian (72), with the help of the closure relation $|f^{\rm ch}\rangle\langle f^{\rm ch}| + |\bar f^{\rm ch}\rangle\langle \bar f^{\rm ch}| = 1$, to the essential "bare" form (Barut and Ziino, 1993)

$$\begin{aligned}\mathcal{L}(|m| \to 0) &= \frac{1}{4}\big[\, i\, (\bar\chi_f \gamma^\mu \partial_\mu \chi_f \,+\, \bar\chi_{\bar f}\, \gamma^\mu \partial_\mu \chi_{\bar f}) \,+\, \text{H.c.}\big] \\ &= \mathcal{L}_{\rm L} \,+\, \mathcal{L}_{\rm R}, \end{aligned} \tag{73}$$

where

$$\mathcal{L}_{\rm L} \equiv \frac{1}{4}(\, i\, \bar\chi_f\, \gamma^\mu \partial_\mu \chi_f \,+\, \text{H.c.})\,, \qquad \mathcal{L}_{\rm R} \equiv \frac{1}{4}(\, i\, \bar\chi_{\bar f}\, \gamma^\mu \partial_\mu \chi_{\bar f} \,+\, \text{H.c.}) \tag{74}$$

(here the subscripts L = "left-handed" and R = "right-handed" clearly have strict covariant meanings, just related to the two chirality signs). The P-invariance property of (73), as well as of the *single* Lagrangians (74), can be inferred again from (49):

$$P : \bar\chi\, \gamma^\mu \partial_\mu \chi \;\longrightarrow\; \bar\chi\, \gamma^0 \gamma^\mu \partial^\mu \gamma^0 \chi = \bar\chi\, \gamma^\mu \partial_\mu \chi. \tag{75}$$

Likewise, recalling the χ-field normalizations in (24), one may suitably introduce a "whole" *fermion–antifermion* covariant massless field

$$\psi(x) \equiv \psi_\pm(x) = \frac{1}{\sqrt{2}}[\pm\chi_f(x) + \chi_{\bar f}(x)], \tag{76}$$

such that

$$\begin{aligned} \frac{1}{\sqrt{2}}\chi_f(x) = \pm X_{\rm L}\psi(x), &\quad \frac{1}{\sqrt{2}}\chi_{\bar f}(x) = X_{\rm R}\psi(x) \\ X_{\rm L} \equiv \frac{1}{2}(1-\gamma^5), &\quad X_{\rm R} \equiv \frac{1}{2}(1+\gamma^5), \end{aligned} \tag{77}$$

and apparently recast (73) as an overall "Dirac" Lagrangian, in terms of the fields ψ and $\bar\psi$ $(= \psi^\dagger \gamma^0)$ (and of their space–time derivatives):

$$\mathcal{L}(|m| \to 0) = \frac{1}{2}(i\; \bar\psi \gamma^\mu \partial_\mu \psi \,+\, \text{H.c.}). \tag{78}$$

Unlike (72), which is so structured as to work in the whole "dressed" Fock space $\mathcal{F}(|m| \to 0) = \mathcal{F}^\circ(|m| \to 0) \otimes S_{\rm in}$, the simplified Lagrangian (73) can also be made working in $\mathcal{F}^\circ(|m| \to 0)$ alone, now taken as an *effective* Fock space. For such a purpose to be realized, the two operators $P_{\rm in}$ and $C_{\rm cov}$ (originally defined in $S_{\rm in}$) should be redefined by exploiting those effects which may equivalently be ascribed to them in the two-dimensional internal space spanned by the chirality-eigenfield basis $(\chi_f, \chi_{\bar f})$: in view of (26) and (27), this can be accomplished by setting

$$\begin{cases} \chi_f^{(P_{\rm in})} \equiv P_{\rm in}\,\chi_f = \chi_{\bar f}, & \chi_{\bar f}^{(P_{\rm in})} \equiv P_{\rm in}\,\chi_{\bar f} = \chi_f \\ \bar\chi_f^{(P_{\rm in})} \equiv \bar\chi_f\,P_{\rm in} = \bar\chi_{\bar f}, & \bar\chi_{\bar f}^{(P_{\rm in})} \equiv \bar\chi_{\bar f}\,P_{\rm in} = \bar\chi_f \end{cases} \tag{79}$$

and

$$\begin{cases} \chi_f^{(C_{\rm cov})} \equiv C_{\rm cov}\chi_f = -\,\chi_f, & \chi_{\bar f}^{(C_{\rm cov})} \equiv C_{\rm cov}\chi_{\bar f} = +\,\chi_{\bar f} \\ \bar\chi_f^{(C_{\rm cov})} \equiv \bar\chi_f\,C_{\rm cov} = -\,\bar\chi_f, & \bar\chi_{\bar f}^{(C_{\rm cov})} \equiv \bar\chi_{\bar f}\,C_{\rm cov} = +\,\bar\chi_{\bar f} \end{cases} \tag{80}$$

(note that the application $P_{\rm in}\,\chi_f = P_{\rm in}X_{\rm L}\sqrt{2}\,\psi_+$ can be represented in four-spinor space as $\gamma^0 X_{\rm L}\sqrt{2}\,\gamma^0\psi_+$, and so on; note also that both $P_{\rm in}$ and $C_{\rm cov}$ are still assumed to behave like *identity* operators in $\mathcal{F}^\circ$, so that you might just as well adopt the notations $\chi_f^{(P_{\rm in})} \equiv P_{\rm in}^\dagger \chi_f P_{\rm in}$, $\chi_f^{(C_{\rm cov})} \equiv C_{\rm cov}^\dagger \chi_f C_{\rm cov}$, and so on). First of all, one has that according to the well-known general anticommutation rules for a, $a^\dagger$, $a^{\rm h\dagger}$, and $a^{\rm h}$, it must always result $a(\mathbf{p}, \sigma_\mp) \neq a^{\rm h}(\mathbf{p}, \sigma_\mp)$ and $a^\dagger(\mathbf{p}, \sigma_\mp) \neq a^{\rm h\dagger}(\mathbf{p}, \sigma_\mp)$. Hence it can be drawn that (73) will still embody two formally *distinct*, alternative covariant pictures, the former just in terms of (positive and negative energy) *left-handed* massless fermions, and the latter just in terms of (positive and negative energy) *right-handed* massless antifermions. In Dirac's language, such a pair of equivalent descriptions still corresponds, as in the nonzero-mass case, to choosing either "particle" = fermion (and "hole" = antifermion) or "particle" = antifermion (and "hole" = fermion). Both pictures have their respective free Lagrangians, $\mathcal{L}_{\rm L}$ and $\mathcal{L}_{\rm R}$, and these (each with its own types of annihilation and creation operators) are acting in two different covariant subspaces of $\mathcal{F}^\circ(|m| \to 0)$, say, $\mathcal{F}_{\rm L}^\circ$ (for positive- and negative-energy fermions) and $\mathcal{F}_{\rm R}^\circ$ (for positive- and negative-energy antifermions):

$$\mathcal{F}^\circ(|m| \to 0) = \mathcal{F}_{\rm L}^\circ \oplus \mathcal{F}_{\rm R}^\circ. \tag{81}$$

The normally ordered versions of $\mathcal{L}_{\rm L}$ and $\mathcal{L}_{\rm R}$ can be written down as

$$(:\mathcal{L}_{\rm L}:) \;=\; \frac{1}{8}\big[\,i\,\big(\bar\chi_f\,\gamma^\mu\partial_\mu\chi_f \;+\; \bar\chi_{\bar f}^{(\rm h)}\,\gamma^\mu\partial_\mu\chi_{\bar f}^{(\rm h)}\big) \;+\; \text{H.c.}\big] \tag{82}$$

and

$$(:\mathcal{L}_{\rm R}:) \;=\; \frac{1}{8}\big[\,i\,\big(\bar\chi_{\bar f}\,\gamma^\mu\partial_\mu\chi_{\bar f} \;+\; \bar\chi_f^{(\rm h)}\,\gamma^\mu\partial_\mu\chi_f^{(\rm h)}\big) \;+\; \text{H.c.}\big], \tag{83}$$

$\chi^{(\rm h)}$ ($\bar\chi^{(\rm h)}$) differing from χ ($\bar\chi$) just by the substitutions a ($a^\dagger$) $\to a^{\rm h}$ ($a^{\rm h\dagger}$) and $a^{\rm h\dagger}$ ($a^{\rm h}$) $\to a^\dagger$ (a); and with the help of the anticommutation rules for the annihilation and creation operators, it can be shown that (thanks to automatic cancellations of infinities) such Lagrangians are nothing but $\mathcal{L}_{\rm L}$ and $\mathcal{L}_{\rm R}$ themselves as rewritten in *manifest* symmetrized (but no longer manifest covariant) forms:

$$\mathcal{L}_{\rm L} = \;(:\mathcal{L}_{\rm L}:), \quad \mathcal{L}_{\rm R} = \;(:\mathcal{L}_{\rm R}:). \tag{84}$$

On the other hand, to get $\mathcal{F}_{\rm L}^{\circ} \rightleftharpoons \mathcal{F}_{\rm R}^{\circ}$, it is clearly sufficient to apply transformation (79), which is further such that

$$\psi^{(P_{\rm in})} \equiv P_{\rm in}\,\psi = \pm\psi, \quad \bar{\psi}^{(P_{\rm in})} \equiv \psi^{\dagger} P_{\rm in}^{\dagger}\gamma^0 = \pm\bar{\psi}, \tag{85}$$

ψ $(\bar{\psi})$ correspondingly standing for either fermion–antifermion field $\psi_{\pm}$ $(\bar{\psi}_{\pm})$ defined in (76). As a result of (79), one just obtains

$$P_{\rm in} : \mathcal{L}_{\rm L} \rightleftharpoons \mathcal{L}_{\rm R}\,; \tag{86}$$

and the same can be written for the manifest symmetrized forms (82) and (83), provided that (79) is extended to the "hole" fields by means of the substitutions $\chi, \bar{\chi} \rightarrow \chi^{(\rm h)}, \bar{\chi}^{(\rm h)}$. Of course, if the overall Fock space (81) were only allowed for, and the Lagrangian (73) were taken only in terms of ψ and $\bar{\psi}$, then the definitions of parity P and charge conjugation C would turn out to be trivial: in line with (85), one simply has

$$\begin{aligned}\bar{\psi}^{(P)}(x_\nu)\gamma^\mu\partial^\mu\psi^{(P)}(x_\nu) &= \bar{\psi}^{(P_{\rm st})}(x_\nu)\gamma^\mu\partial^\mu\psi^{(P_{\rm st})}(x_\nu)\\ &= \bar{\psi}(x^\nu)\gamma^0\gamma^\mu\partial^\mu\gamma^0\psi(x^\nu)\end{aligned} \tag{87}$$

and

$$\bar{\psi}^{(C)}\gamma^\mu\partial_\mu\psi^{(C)} = \bar{\psi}^{(C_{\rm st})}\gamma^\mu\partial_\mu\psi^{(C_{\rm st})} = \bar{\psi}^{(\rm h)}\gamma^\mu\partial_\mu\psi^{(\rm h)}, \tag{88}$$

where $P_{\rm st}$ is strictly defined by

$$P_{\rm st} : \begin{cases} a(\mathbf{p}, \sigma_{\mp}) \rightarrow a(-\mathbf{p}, \sigma_{\pm}), & a^{\rm h\dagger}(\mathbf{p}, \sigma_{\pm}) \rightarrow -a^{\rm h\dagger}(-\mathbf{p}, \sigma_{\mp})\\ a^{\dagger}(\mathbf{p}, \sigma_{\mp}) \rightarrow a^{\dagger}(-\mathbf{p}, \sigma_{\pm}), & a^{\rm h}(\mathbf{p}, \sigma_{\pm}) \rightarrow -a^{\rm h}(-\mathbf{p}, \sigma_{\mp})\end{cases} \tag{89}$$

and $C_{\rm st}$ by

$$C_{\rm st} : \begin{cases} a(\mathbf{p}, \sigma_{\mp}) \rightarrow a^{\rm h}(\mathbf{p}, \sigma_{\mp}), & a^{\rm h\dagger}(\mathbf{p}, \sigma_{\pm}) \rightarrow a^{\dagger}(\mathbf{p}, \sigma_{\pm})\\ a^{\dagger}(\mathbf{p}, \sigma_{\mp}) \rightarrow a^{\rm h\dagger}(\mathbf{p}, \sigma_{\mp}), & a^{\rm h}(\mathbf{p}, \sigma_{\pm}) \rightarrow a(\mathbf{p}, \sigma_{\pm}).\end{cases} \tag{90}$$

It is easily seen, however, that neither $\mathcal{F}_{\rm L}^{\circ}$ nor $\mathcal{F}_{\rm R}^{\circ}$ (but only their sum) is an invariant space with respect to the individual operations (89) and (90). So, to make sure that P and C are defined also *within* each single space $\mathcal{F}_{\rm L,R}^{\circ}$, it *cannot* merely be assumed that $P = P_{\rm st}$ and $C = C_{\rm st}$: in full analogy with the "dressed" formalism, it must rather be put $P = P_{\rm in}P_{\rm st}$ (or $P = P_{\rm in}P_{\rm ex}$) and $C = P_{\rm in}C_{\rm st}$, with P being still reducible to $P_{\rm st}$ (or $P_{\rm ex}$) and C to $C_{\rm st}$, if applied to the field ψ $(\bar{\psi})$ "as a whole." One gets, for instance,

$$\begin{aligned}\bar{\chi}_f^{(P)}(x_\nu)\gamma^\mu\partial^\mu\chi_f^{(P)}(x_\nu) &= \bar{\chi}_{\bar{f}}^{(P_{\rm st})}(x_\nu)\gamma^\mu\partial^\mu\chi_{\bar{f}}^{(P_{\rm st})}(x_\nu)\\ &= \bar{\chi}_f(x^\nu)\gamma^0\gamma^\mu\partial^\mu\gamma^0\chi_f(x^\nu)\end{aligned} \tag{91}$$

and

$$\bar{\chi}_f^{(C)}\gamma^\mu\partial_\mu\chi_f^{(C)} = \bar{\chi}_{\bar{f}}^{(C_{\rm st})}\gamma^\mu\partial_\mu\chi_{\bar{f}}^{(C_{\rm st})} = \bar{\chi}_{\bar{f}}^{(\rm h)}\gamma^\mu\partial_\mu\chi_{\bar{f}}^{(\rm h)}, \tag{92}$$

where it has been allowed for (79) (and its "hole" counterpart) and for the fact that

$$\begin{cases} \chi_{\bar{f}}^{(P_{\rm st})}(x_\nu) = \frac{1}{\sqrt{2}}(1+\gamma^5)\psi^{(P_{\rm st})}(x_\nu) = \frac{1}{\sqrt{2}}(1+\gamma^5)\gamma^0\psi(x^\nu) = \gamma^0\chi_f(x^\nu) \\ \bar{\chi}_{\bar{f}}^{(P_{\rm st})}(x_\nu) = \frac{1}{\sqrt{2}}\bar{\psi}^{(P_{\rm st})}(x_\nu)(1-\gamma^5) = \frac{1}{\sqrt{2}}\bar{\psi}(x^\nu)\gamma^0(1-\gamma^5) = \bar{\chi}_f(x^\nu)\gamma^0. \end{cases} \tag{93}$$

The invariance of the single Lagrangians $\mathcal{L}_{\rm L,R}$ under P $(= P_{\rm in}P_{\rm st})$ is still obtained as in (75), and the invariance of them under C $(= P_{\rm in}C_{\rm st})$ is made manifest once they are taken in their respective symmetrized forms (82) and (83). By comparing such forms, one can see as well that applying $C_{\rm st}$ to either of them yields indeed the same net result as applying $P_{\rm in}$ to it:

$$C_{\rm st} : (:\mathcal{L}_{\rm L}:) \rightleftharpoons (:\mathcal{L}_{\rm R}:). \tag{94}$$

In this sense, transformation (90) may also be regarded as an *effective* (non-covariant) representation of $P_{\rm in}$ itself, say, $P_{\rm in}^{(\rm eff)}$, acting in $\mathcal{F}^\circ(|m| \to 0)$. Such an outcome, $C_{\rm st} = P_{\rm in}^{(\rm eff)}$, can be suitably shown up by setting

$$\begin{cases} a(\mathbf{p},\sigma_-) \equiv a(\mathbf{p},\sigma_-\,;L), \quad a^{\rm h\dagger}(\mathbf{p},\sigma_+) \equiv a^\dagger(\mathbf{p},\sigma_+\,;L) \\ a^\dagger(\mathbf{p},\sigma_-) \equiv a^\dagger(\mathbf{p},\sigma_-\,;L), \quad a^{\rm h}(\mathbf{p},\sigma_+) \equiv a(\mathbf{p},\sigma_+\,;L) \end{cases} \tag{95}$$

for the *left-handed* picture, and

$$\begin{cases} a(\mathbf{p},\sigma_+) \equiv a(\mathbf{p},\sigma_+\,;R), \quad a^{\rm h\dagger}(\mathbf{p},\sigma_-) \equiv a^\dagger(\mathbf{p},\sigma_-\,;R) \\ a^\dagger(\mathbf{p},\sigma_+) \equiv a^\dagger(\mathbf{p},\sigma_+\,;R), \quad a^{\rm h}(\mathbf{p},\sigma_-) \equiv a(\mathbf{p},\sigma_-\,;R) \end{cases} \tag{96}$$

for the *right-handed* one: transformation (90) is thus made an explicit $L \rightleftharpoons R$ interchange operation. From comparing (90) with both (95) and (96), it can, however, be immediately inferred that at variance with the massive-chiral-field case, the only effect really produced by either $P_{\rm in}$ or $C_{\rm st}$ on a massless spin-$\frac{1}{2}$ fermion (antifermion) is to transform it, as described in one picture, into *itself* as described in the other picture: the point is that *no* helicity flip is involved, and so the "transformed" (massless) spin-$\frac{1}{2}$ particle cannot look really changed. In the limiting case of zero-mass fermions, one has indeed that the actual operation of particle $\rightleftharpoons$ antiparticle conjugation cannot be disjoined from parity, or better, from "external" parity. Suitable *effective* representations may also be found, accordingly, for C $(= P_{\rm in}C_{\rm st})$ and P $(= P_{\rm in}P_{\rm st})$; these, $C^{(\rm eff)}$ and $P^{(\rm eff)}$, hold both in either *single* Fock space $\mathcal{F}^\circ_{\rm L,R}$ and are such that $C^{(\rm eff)} = 1$ and $P^{(\rm eff)} = C^{(\rm eff)}P^{(\rm eff)} = C_{\rm st}P_{\rm st} = (CP)_{\rm st}$. For instance, it can easily be seen that the general prescription (75), as applied to (82), globally corresponds to

$$P^{(\rm eff)} : \begin{cases} a(\mathbf{p},\sigma_-) \to a^{\rm h}(-\mathbf{p},\sigma_+), \quad a^{\rm h\dagger}(\mathbf{p},\sigma_+) \to -a^\dagger(-\mathbf{p},\sigma_-) \\ a^\dagger(\mathbf{p},\sigma_-) \to a^{\rm h\dagger}(-\mathbf{p},\sigma_+), \quad a^{\rm h}(\mathbf{p},\sigma_+) \to -a(-\mathbf{p},\sigma_-). \end{cases} \tag{97}$$

Within this parity-symmetric framework − which, nevertheless, paradoxically admits only a left-handed (right-handed) field solution for any zero-mass fermion (antifermion) – one has that the two "covariantly conjugated" massless chiral fields (77) (the former pertaining to a positive- and negative-energy fermion, the latter to a positive- and negative-energy antifermion) are nothing but *chirality conjugated* fields, each just *reproducing* the "missing counterpart" of the other: indeed, the overall fermion–antifermion field (76) is formally structured the same as a "whole" Dirac field with "chiral projections" (77). An explicit use of Weyl's γ-matrix representation thus leads to a *natural* two-component scheme for a massless spin-$\frac{1}{2}$ fermion, and to a pure *mirror* analogue for the corresponding antifermion, with *no* chiral contribution really missing on the whole. This is a scheme strictly precluding a left-right symmetric electroweak theory like the one known in the literature (Mohapatra and Pati, 1975). This may also be said to be a *universal* two-component theory, in the sense that, far from being an *ad hoc* theory for neutrinos (Salam, 1957; Weyl, 1929), it should quite generally apply to *whatever massless spin-$\frac{1}{2}$ particle*. On the basis of such a scheme, the following nonstandard conclusion can be drawn: *Just applying helicity reversal to a massless (and only left-handed) spin-$\frac{1}{2}$ fermion does already mean turning it into its (only right-handed) antifermion, without any real mirror-symmetry failure involved.* This statement is closely related to the effective result $P = CP$ expressed by (97), with $C = 1$ somehow reminding us of the Majorana neutrino model (Majorana, 1937). If we take also account of the fact that the two fields (77) form just a pair of C_{cov} (i.e., *scalar-charge conjugation*) *eigenfields*, we may then, after all, fully establish that *a massless spin-$\frac{1}{2}$ particle and its antiparticle are in themselves the pure mirror images of each other*.

The property of global gauge invariance of (73) under the chiral group $\text{U}^{(\gamma^5)}(1)$, with a generator suitably defined as

$$Q^{(\gamma^5)} = \pm|q^{\text{ch}}|\,(-\gamma^5), \tag{98}$$

clearly yields the conserved *free fermion–antifermion covariant chiral current*

$$J^{\text{ch}(\gamma^5)} \equiv \bar{\psi}\gamma^\mu Q^{(\gamma^5)}\psi, \tag{99}$$

where

$$\bar{\psi}\gamma^\mu Q^{(\gamma^5)}\psi = \frac{1}{2}\,[\,q^{\text{ch}}\,\bar{\chi}_f\gamma^\mu\chi_f \;+\; (-q^{\text{ch}})\,\bar{\chi}_{\bar{f}}\,\gamma^\mu\chi_{\bar{f}}\,] \tag{100}$$

($q^{\text{ch}} = \pm|q^{\text{ch}}|$). From (77) and (80), it is immediate to see that $Q^{(\gamma^5)}$ is just a representation of the charge $Q^{\text{ch}} = \pm|q^{\text{ch}}|(-C_{\text{cov}})$ in four-spinor space. The "dressed" complete version of (99) reads

$$J^{\text{ch}(\gamma^5)} \equiv \bar{\psi}\gamma^\mu Q^{(\gamma^5)}\psi\,(|f^{\text{ch}}\rangle\langle f^{\text{ch}}| \;+\; |\bar{f}^{\text{ch}}\rangle\langle\bar{f}^{\text{ch}}|) \tag{101}$$

and is manifestly invariant with respect to P_{in} as taken in its original definition (19). The current $J^{\text{ch}(\gamma^5)}$ carries the *pseudoscalar* charge $Q^{(\gamma^5)}$ (such that $\gamma^0 Q^{(\gamma^5)} = -Q^{(\gamma^5)}\gamma^0$) of which the two four-spinor fields χ_f and $\chi_{\bar{f}}$ are conjugated eigenspinors with eigenvalues $+q^{\text{ch}}$ and $-q^{\text{ch}}$. It therefore behaves under P just like an *axial-vector* current:

$$P : \bar{\psi}(x^\nu)\gamma^\mu Q^{(\gamma^5)}\psi(x^\nu) \longrightarrow \bar{\psi}^{(P)}(x_\nu)\gamma^\mu Q^{(\gamma^5)}\psi^{(P)}(x_\nu)$$

$$= \bar{\psi}(x^\nu)\gamma^0\gamma^\mu Q^{(\gamma^5)}\gamma^0\psi(x^\nu). \tag{102}$$

The two individual (fermion and antifermion) sectors of $J^{\text{ch}(\gamma^5)}$ are themselves axial-vector currents. This can be seen by simply substituting (77): if the $Q^{(\gamma^5)}$ eigenvalues are made explicit, then, in view of (75), the result is

$$P : \chi_f, \bar{\chi}_f \to \gamma^0\chi_f, \bar{\chi}_f\gamma^0; \quad \chi_{\bar{f}}, \bar{\chi}_{\bar{f}} \to \gamma^0\chi_{\bar{f}}, \bar{\chi}_{\bar{f}}\gamma^0; \quad q^{\text{ch}} \rightleftharpoons -q^{\text{ch}} \tag{103}$$

and automatically allows for the fact that both q^{ch} and $-q^{\text{ch}}$ are *pseudoscalars* (and must be taken with inverted signs in the same way as space coordinates). The corresponding behavior of $J^{\text{ch}(\gamma^5)}$ under P_{in} is dictated by the prescription (70); hence,

$$P_{\text{in}} : \bar{\chi}_f\gamma^\mu Q^{(\gamma^5)}\chi_f \rightleftharpoons \bar{\chi}_{\bar{f}}\gamma^\mu Q^{(\gamma^5)}\chi_{\bar{f}}\,, \tag{104}$$

so that, using Eqs. (85), one may write

$$P_{\text{in}} : \bar{\psi}\gamma^\mu Q^{(\gamma^5)}\psi \longrightarrow \bar{\psi}^{(P_{\text{in}})}\gamma^\mu Q^{(\gamma^5)}\psi^{(P_{\text{in}})} = \bar{\psi}\gamma^\mu Q^{(\gamma^5)}\psi. \tag{105}$$

Both (104) and (105) could also have been straightforwardly deduced from applying the requirement of covariance in the internal space defined by the field basis $(\chi_f, \chi_{\bar{f}})$. Due to its symmetrized fermion–antifermion original structure, the whole current $J^{\text{ch}(\gamma^5)}$ can naturally be recast (owing to an automatic cancellation of infinities) into a (manifest) normally-ordered form,

$$J^{\text{ch}(\gamma^5)} = \big(: J^{\text{ch}(\gamma^5)} :\big), \tag{106}$$

which consists of the sum of two symmetrized "particle–hole" currents, with "particle" = fermion (and "hole" = antifermion) and with "particle" = antifermion (and "hole" = fermion) respectively. Thanks to the fact that $\gamma^\mu Q^{(\gamma^5)} = -Q^{(\gamma^5)}\gamma^\mu$, either one of these (mutually *exclusive*) currents is expressible as

$$\frac{1}{2}\big(: \bar{\chi}\,\gamma^\mu Q^{(\gamma^5)}\chi :\big) = \frac{1}{4}\big[\bar{\chi}\,\gamma^\mu Q^{(\gamma^5)}\chi + \bar{\chi}^{(C)}\gamma^\mu Q^{(\gamma^5)}\chi^{(C)}\big], \tag{107}$$

where C $(= C_{\text{st}}P_{\text{in}})$ is the ordinary charge-conjugation operation as represented in the Fock space (2), and where (still using the same symbol C also to denote the C-matrix) one has that $\chi^{(C)} = C\widetilde{\bar{\chi}}^\dagger$ and $\bar{\chi}^{(C)} = \chi^{\dagger(C)}\gamma^0 = \widetilde{\chi}C^\dagger\gamma^0$. So, since $\chi^{(C)}$ $(\bar{\chi}^{(C)})$ correspondingly stands for the "ordinary charge-conjugate" of χ $(\bar{\chi})$, each current (107) may indeed be said to be *manifestly C-invariant* (just as it more familiarly happens for the normally-ordered version of the Dirac-like "whole" current $J^{\mathrm{ch}(\gamma^5)\mu} = \bar{\psi}\gamma^\mu Q^{(\gamma^5)}\psi$). This exactly counterbalances the effect due to P, so that the overall intrinsic behavior of a current like (107) under CP will be still the same as the one of a scalar-charge (vector) current.

Actually, the two massless fields χ_f and $\chi_{\bar{f}}$ are naturally prevented from being *scalar-charge* conjugated eigenfields (with opposite eigenvalues), since an overall axial-vector current like (99), carrying a true charge like (98), would be met with again. At the origin of such an outcome, there is clearly the fact that pseudoscalar and scalar charges anticommute (this basic property may further be expected to underlie the well-known *axial anomaly* occurring in the limit as $m \to 0$). Yet, if one is dealing with the only square magnitude, say, $|q|^2$, of a scalar charge, then, strictly speaking, one may always think also of a conserved free current having the form

$$J^{\mathrm{ch}} \equiv \pm|q|\,\bar{\psi}\gamma^\mu\psi = \pm|q|\,\frac{1}{2}\,(\bar{\chi}_f\gamma^\mu\chi_f + \bar{\chi}_{\bar{f}}\,\gamma^\mu\chi_{\bar{f}}), \tag{108}$$

where the c-number $\pm|q|$ is the *same* for χ_f *as* for $\chi_{\bar{f}}$ and therefore stands for a mere *root* of $|q|^2$ (with a purely conventional sign). Setting $q = \pm|q|$ and $Q = qP_{\mathrm{in}}$, and recalling Eqs. (85) and (76), one should more rigorously write

$$J^{\mathrm{ch}} \equiv \bar{\psi}\gamma^\mu Q\psi = \bar{\psi}Q\gamma^\mu\psi = \pm q\,\bar{\psi}\gamma^\mu\psi, \tag{109}$$

Q being the scalar charge in question, formally having the "whole" field ψ ($\bar{\psi}$) as an eigenfield:

$$Q\psi = Q\psi_\pm = \pm q\psi_\pm, \quad \bar{\psi}Q = \bar{\psi}_\pm Q = \pm q\bar{\psi}. \tag{110}$$

Since, according to (79), one has, e.g., $\chi_f^\dagger Q\,\chi_f = q\,\chi_f^\dagger\chi_{\bar{f}} = q\,\chi_{\bar{f}}^\dagger\chi_f = 0$, it is evident that both the fermion and the antifermion can only be assigned an expectation value $\langle Q\rangle = 0$, which corresponds to a *maximal uncertainty* as regards the actual (relative) sign taken by Q. The new current J^{ch} is indeed a *vector* current (not normally ordered yet) which can be derived starting from the invariance of (78) under the group of global gauge transformations $\psi \to e^{iQ\alpha}\psi$, $\bar{\psi} \to \bar{\psi}e^{-iQ\alpha}$, generated by Q (α being a constant real angle). The symmetrized "particle–hole" version of it,

$$(:J^{\mathrm{ch}}:) = \pm|q|\,\frac{1}{2}\big(\bar{\psi}\gamma^\mu\psi - \bar{\psi}^{(\mathrm{h})}\gamma^\mu\psi^{(\mathrm{h})}\big), \tag{111}$$

is such that

$$C\,(C_{\mathrm{st}}) : (:J^{\mathrm{ch}}:) \longrightarrow -(:J^{\mathrm{ch}}:), \tag{112}$$

where C (not affecting helicities) is causing still no actual fermion $\rightleftharpoons$ antifermion conjugation, but only an overall $\bar{\psi}\gamma^\mu\psi \rightleftharpoons \bar{\psi}^{(\mathrm{h})}\,\gamma^\mu\psi^{(\mathrm{h})}$ interchange, equivalent to a net change in sign of the single $|q|^2$ root associated with both the fermion and the antifermion. Since $C_{\text{cov}}Q = -QC_{\text{cov}}$ and $\bar{\psi}^{(C_{\text{cov}})}\gamma^{\mu}\psi^{(C_{\text{cov}})} = \bar{\psi}\gamma^{\mu}\psi$, there is also a strict covariant analogue of (112), which reads

$$C_{\text{cov}} : J^{\text{ch}} \longrightarrow -J^{\text{ch}}. \tag{113}$$

This implies

$$C_{\text{cov}} : \pm|q| \longrightarrow \mp|q| \tag{114}$$

when Q is replaced by $\pm|q|$ as in (108).

On the basis of all that, it can be rigorously established that the unusual parity-symmetry in hand, paradoxically available for a pure left-handed massless spin-$\frac{1}{2}$ fermion and its (pure right-handed) antiparticle, is to be traced back just to their essential being a pair of *pseudoscalar-charge conjugated eigenstates* (which, by nature, are transformed into each other under parity itself). As pointed out by (108), two such particles are allowed to bear also (nonzero) scalar charges, even though definite only in magnitude and *maximally indefinite in sign*: given, e.g., one of these charges, of square magnitude $|q|^2$, they may, in principle, be assigned only a (conventionally chosen) $|q|^2$ root – either $+|q|$ or $-|q|$ for *them both* – which clearly gives no sort of information on the (relative) sign of such a charge. The most immediate consequence is that an *electrically charged* pair of covariantly conjugated (fermion and antifermion) massless fields (77) can just be said to carry the "same" electric charge (suitably expressed by either root $\pm|q| = \pm|e|$). This peculiar outcome will indeed turn out to play a crucial role for consistently revisiting the $\text{SU}(2)_{\text{L}}\otimes\text{U}(1)$ electroweak scheme: thanks to it, the "spurious" *fermion* (right-handed) massless isosinglets may survive as taken just for regular *antifermion* $\text{SU}(2)_{\text{L}}$-isosinglets.

To sum up, the massless spin-$\frac{1}{2}$ fermion universally emerging from the spontaneous two-component theory above can be depicted like this: it should typically behave as a pure *pseudoscalar*-charge eigenstate, which is endowed with only *one* helicity freedom degree and can at most carry scalar charges subject to *a maximal uncertainty in sign*. This turns out to be the natural zero-mass limit of the generalized fermion model in hand, founded on the coexistence of two *anticommuting*, scalar and pseudoscalar, charge varieties.

5. WEAK-ISOSPIN COMPONENTS WITH PSEUDOSCALAR BEHAVIORS UNDER PARITY

In the light of such premises, one can now try to reformulate the electroweak scheme (Glashow, 1961; Salam, 1968; Weinberg, 1967) on deeper theoretical grounds. To begin with, let $\mathbf{T}^{\text{w}}$ denote the *weak isospin* one-particle operator. In the standard (primary) version without masses, $\mathbf{T}^{\text{w}}$ is just *assumed* to act selectively on fermion isodoublets being *left-handed*. This appears therein to be an *ad hoc* input, devoid of physical motivations, but needed to allow for the "maximally P-violating" phenomenology (Ambler *et al.*, 1957; Lee and Yang, 1956). The final consequences, after mass generation, seem to be rather perplexing from a strict theoretical viewpoint: those which (according to standard views) have become two mere "chiral projections" of one and the same (Dirac) fermion field are curiously left assigned to two *different* weak-isospin representations. The reason for such an *accommodation* quite vanishes on going over to the generalized formalism in hand, with the above-seen *natural* two-component fermion model as a zero-mass limit. This formalism can just *define* a "chiral" particle (regardless of experience): it should look *diametrically opposed* to a "Dirac" particle, or better, like a pseudoscalar-charge eigenstate with null scalar-charge expectation values. When mass is already present, a "Dirac" and a "chiral" particle are nothing but two *complementary* (though mutually exclusive) *dynamical manifestations* of one and the same fermion (or antifermion). When, on the contrary, mass is still to appear, no "Dirac" particles can yet be found: the only massless-fermion type allowed is just a (left-handed) "chiral" fermion, and the pure (right-handed) mirror image of it just corresponds to its antiparticle. So, it is massless fermions (antifermions) themselves that are, by nature, *only* left-handed (right-handed); and this explains why $\mathbf{T}^{\mathrm{w}}$ should act merely on fermion (antifermion) isodoublets being left-handed (right-handed).

In line with what has been argued in the previous section, the third component of $\mathbf{T}^{\mathrm{w}}$ is now required to behave as a *pseudoscalar* charge under parity (if its eigenvalues are to correspond still to *massless* primary eigenstates). The same, of course, must hold for each one of the remaining $\mathbf{T}^{\mathrm{w}}$ components. Let then $\mathbf{T}^{\mathrm{w}}$ be suitably redefined, by analogy with (98), as

$$\mathbf{T}^{\mathrm{w}} = \mathbf{T}\,(-\gamma^5), \tag{115}$$

where $\mathbf{T}$ stands for a usual one-particle isospin operator, whose components, as taken in the fundamental representation, are $T_i = \tau_i/2$ $(i = 1, 2, 3)$ $(\tau_1, \tau_2$, and τ_3 being the Pauli matrices). Evidently, due to the presence of $(-\gamma^5)$, the weak isospin (115) *as such* cannot generate any SU(2) transformation group; from it, two *distinct* (left- and right-handed) SU(2) generators can rather be obtained, provided that either one of the $(-\gamma^5)$ eigenvalues is directly involved. More precisely, let us put

$$\mathbf{T}^{\mathrm{w}} = \mathbf{T}_{\mathrm{L}} + \mathbf{T}_{\mathrm{R}}, \tag{116}$$

where

$$\mathbf{T}_{\mathrm{L}} = \mathbf{T}^{\mathrm{w}} X_{\mathrm{L}} = \mathbf{T} X_{\mathrm{L}}, \quad \mathbf{T}_{\mathrm{R}} = \mathbf{T}^{\mathrm{w}} X_{\mathrm{R}} = (-\mathbf{T}) X_{\mathrm{R}}, \tag{117}$$

X_{L} and X_{R} being the chirality projection operators quoted in (77), and where we shall have that $\mathbf{T}^{\mathrm{w}} = \mathbf{T}_{\mathrm{L}}$ or $\mathbf{T}^{\mathrm{w}} = \mathbf{T}_{\mathrm{R}}$ according to whether $\mathbf{T}^{\mathrm{w}}$ is acting on left- or right-handed chiral fields. It is immediate then to see that the single operators $\mathbf{T}_{\mathrm{L}}$ and $\mathbf{T}_{\mathrm{R}}$ are naturally able to generate two *distinct* SU(2) groups, say, $\mathrm{SU(2)_L}$ and $\mathrm{SU(2)_R}$, whose fundamental representations – the former applying to (left-handed) *fermion* chiral fields, and the latter to (right-handed) *antifermion* ones – are marked by the *effective* isospin generators $\mathbf{T}$ and $(-\mathbf{T})$, respectively. Such groups should be thought of as being involved only *in turn*, depending on whether a fermion or antifermion *covariant* picture is being adopted. So, at the zero-mass primary stage, one may equally deal either with (fermion) $\mathrm{SU(2)_L}$ isodoublets, of the *lepton* type

$$D_{\mathrm{L}}^{(\ell)} = \begin{pmatrix} \chi_\nu \\ \chi_\ell \end{pmatrix} \qquad (\nu \equiv \nu_\ell; \qquad \ell = e, \mu, \tau) \tag{118}$$

and of the *quark* type

$$D_{\mathrm{L}}^{(\mathrm{q})} = \begin{pmatrix} \chi_{\mathrm{p}} \\ \chi_{\mathrm{n}} \end{pmatrix} \qquad (\mathrm{p} = \mathrm{u, c, t}; \qquad \mathrm{n} = \mathrm{d, s, b}), \tag{119}$$

or with (antifermion) $\mathrm{SU(2)_R}$ isodoublets, of the *antilepton* type

$$D_{\mathrm{R}}^{(\bar{\ell})} = \begin{pmatrix} \chi_{\bar{\nu}} \\ \chi_{\bar{\ell}} \end{pmatrix} \tag{120}$$

and of the *antiquark* type

$$D_{\mathrm{R}}^{(\bar{\mathrm{q}})} = \begin{pmatrix} \chi_{\bar{\mathrm{p}}} \\ \chi_{\bar{\mathrm{n}}} \end{pmatrix}, \tag{121}$$

where

$$T_3^w D_{\mathrm{L}} = T_{3\mathrm{L}} D_{\mathrm{L}} = \frac{1}{2}\tau_3 D_{\mathrm{L}}, \quad T_3^w D_{\mathrm{R}} = T_{3\mathrm{R}} D_{\mathrm{R}} = \frac{1}{2}(-\tau_3) D_{\mathrm{L}}. \tag{122}$$

The single members of any fermion isodoublet D_{L} and of its antifermion version D_{R} are the mere *chirality-conjugate* counterparts of each other, as according to (77). Each isodoublet D_{L} (D_{R}) will then be mapped onto its "covariant charge-conjugate" isodoublet D_{R} (D_{L}) by

$$P_{\mathrm{in}} : D_{\mathrm{L}}^{(\ell)} \rightleftharpoons D_{\mathrm{R}}^{(\bar{\ell})}, \quad D_{\mathrm{L}}^{(\mathrm{q})} \rightleftharpoons D_{\mathrm{R}}^{(\bar{\mathrm{q}})} \tag{123}$$

with P_{in} acting as in (79); and the definitions of D_{L} and D_{R} may look *symmetrical*, just because one correspondingly has

$$P_{\mathrm{in}} : \mathrm{SU(2)_L} \rightleftharpoons \mathrm{SU(2)_R}. \tag{124}$$

In this way, two covariant SU(2) pictures being the pure *mirror* (i.e., *chirality-conjugate*) counterparts of each other are equally allowable, the former manifestly dealing with (positive- and negative-energy) massless fermions, and the latter with (positive- and negative-energy) massless antifermions. These are also two *mutually exclusive* SU(2) pictures, in the sense that the individual isodoublet members of one picture are mere *isosinglets* in the other picture. The overall observance of left–right symmetry is indeed shown up by the fact that there are in addition neither right-handed massless fermions nor left-handed massless antifermions *as such*, treated asymmetrically with respect to their own mirror partners.

To summarize, for the zero-mass primary stage under consideration, a *twofold* – either $SU(2)_L$ or $SU(2)_R$ – variety of weak-isospin representations is now automatically available, according to whether a covariant picture manifestly in terms of (positive- and negative-energy) fermions or antifermions is being adopted. This peculiar feature directly follows from the new weak-isospin definition (115) and should therefore be expected to survive even after mass appearance. Of course, in a *pure* SU(2) local gauge model, no isosinglets could ever be involved. Different is instead the case of an SU(2)⊗U(1) model (like the electroweak one) in which also a special subgroup generated by a *scalar* charge Q takes an active part. Within it, "whole" left–right currents of the type (109) (needed to diagonalize Q itself) should be involved as well, which will implicitly contain either $SU(2)_L$ (antifermion) or $SU(2)_R$ (fermion) *isosinglet* contributions, just miming those ones due to the *ad hoc* (right-handed fermion and left-handed antifermion) standard isosinglets.

The new basic general pattern of free Lagrangian for an electroweak scheme will be provided by a strict covariant form like

$$\mathcal{L} = \frac{1}{4}\big[\, i\,(\bar{D}_{\rm L}\gamma^{\mu}\partial_{\mu} D_{\rm L} \,+\, \bar{D}_{\rm R}\gamma^{\mu}\partial_{\mu} D_{\rm R}) \,+\, \text{H.c.}\big], \tag{125}$$

with

$$D_{\rm L} = \begin{pmatrix} \chi_{\rm a} \\ \chi_{\rm b} \end{pmatrix}, \qquad D_{\rm R} = \begin{pmatrix} \chi_{\bar{\rm a}} \\ \chi_{\bar{\rm b}} \end{pmatrix} \tag{126}$$

($\bar{\rm a}$, $\bar{\rm b}$ denoting the antiparticles of a, b). This starting Lagrangian, invariant under P as well as under $P_{\rm in}$, is manifestly *left–right symmetric* and may in principle be suitable for *both* alternative covariant pictures (each one with a complete energy spectrum) in terms of either $SU(2)_L$ (fermion) or $SU(2)_R$ (antifermion) isodoublets. If we want in particular to select the $SU(2)_L$ (fermion) picture, we should appropriately rewrite (125) in the "asymmetrical" form

$$\mathcal{L}_{\rm (L)} = \frac{1}{4}[\, i\,(\bar{D}_{\rm L}\gamma^{\mu}\partial_{\mu} D_{\rm L} \,+\, \bar{\chi}_{\bar{a}}\,\gamma^{\mu}\partial_{\mu}\chi_{\bar{a}} \,+\, \bar{\chi}_{\bar{b}}\,\gamma^{\mu}\partial_{\mu}\chi_{\bar{b}}) \,+\, \text{H.c.}], \tag{127}$$

where $\chi_{\bar{a}}$ ($\bar{\chi}_{\bar{a}}$) and $\chi_{\bar{b}}$ ($\bar{\chi}_{\bar{b}}$) enter as mere $SU(2)_L$ *isosinglets*. This evidently leads us to a *spontaneous breaking* of $P_{\rm in}$ symmetry: applying $P_{\rm in}$ to $\mathcal{L}_{\rm (L)}$ will mean turning it into

$$\mathcal{L}_{\rm (R)} = \frac{1}{4}[\, i\,(\bar{D}_{\rm R}\gamma^{\mu}\partial_{\mu} D_{\rm R} \,+\, \bar{\chi}_{a}\,\gamma^{\mu}\partial_{\mu}\chi_{a} \,+\, \bar{\chi}_{b}\,\gamma^{\mu}\partial_{\mu}\chi_{b}) \,+\, \text{H.c.}], \tag{128}$$

where $\mathcal{L}_{(R)}$ is nothing but $\mathcal{L}$ as likewise re-expressed in an "asymmetrical" form just pertaining to the $SU(2)_R$ (antifermion) picture. More generally, one has that the P_{in}-invariance property originally shown by $\mathcal{L}$ would be "hidden" as follows:

$$P_{in} : \mathcal{L} = \mathcal{L}_{(L)} \rightleftharpoons \mathcal{L} = \mathcal{L}_{(R)}. \tag{129}$$

If the local gauge model to be built were a pure SU(2) one, generated by *pseudoscalar* charges only, then the isosinglet sector could be taken away from either (127) or (128), since it would be both redundant and dynamically irrelevant. This sector cannot be ignored, on the contrary, for a local gauge scheme just like the electroweak one, in which (at least) one *scalar* charge is involved, being responsible for the covariant generation of "whole" left–right U(1) gauge couplings.

6. PARITY-INVARIANT GAUGE SCHEME FOR LEPTONS AND QUARKS IN THE ABSENCE OF MASSES

Let us suppose, as usual, both leptons and quarks to be initially massless, and let us apply to them the two-component universal model of Section 4. At variance with what has been believed so far, the neutrino and the antineutrino of any given lepton family ℓ $(= e, \mu, \tau)$ are then to be *P-symmetrically* interpreted as just a pair of *pseudoscalar*-charge conjugated particles being eigenstates of scalar-charge conjugation and intrinsically amounting to pure *mirror images* of each other. The same should hold for the so-called "charged" leptons, along with their own antileptons, and for the two quark types p (= u, c, t) and n (= d, s, b), along with their own antiquarks: as long as they are taken massless, they may at most carry, besides well-defined pseudoscalar charges, only scalar charges looking *maximally indefinite* in sign (if not zero). In short, a new basic feature should mark, without distinctions, all initial leptons and quarks: *their natural being pseudoscalar-charge eigenstates with null scalar-charge expectation values*. As already pointed out, this model has a *pure theoretical* origin: it is the zero-mass extreme consequence of a generalized fermion quantum field formalism which, by means of the internal transformation (25), is able to define a *covariant* pair of "pseudoscalar-charge conjugated" massive *chiral* fields, χ_f and $\chi_{\bar{f}}$, on the same footing as a *covariant* pair of "scalar-charge conjugated" massive *Dirac* fields, ψ_f and $\psi_{\bar{f}}$, in a framework spontaneously including two *anticommuting*, scalar and pseudoscalar, charge varieties.

On dealing with massless leptons and quarks so conceived, we can just exploit the general covariant fermion–antifermion free Lagrangian $\mathcal{L}(|m| \to 0)$ as expressed in the simplified form (73) with (77) understood:

$$\mathcal{L}(|m| \to 0) = \mathcal{L}_{f\bar{f}}, \quad (f = \nu_\ell, \ell, \text{p}, \text{n}). \tag{130}$$

To begin with, for any individual family of neutrino–antineutrino pairs, we shall have (Barut and Ziino, 1993)

$$\mathcal{L}_{\nu\bar{\nu}} = \frac{1}{4}[\, i\, (\bar{\chi}_\nu \gamma^\mu \partial_\mu \chi_\nu \, + \, \bar{\chi}_{\bar{\nu}}\, \gamma^\mu \partial_\mu \chi_{\bar{\nu}}) \, + \, \text{H.c.}], \quad (\nu \equiv \nu_\ell), \tag{131}$$

χ_ν ($\bar{\chi}_\nu$) and $\chi_{\bar{\nu}}$ ($\bar{\chi}_{\bar{\nu}}$) formally being, in view of (77), the mere *chirality conjugates* of each other. A Lagrangian like (131) yields indeed a *natural* neutrino two-component scheme, with no parity violation being really involved: the left- and right-handed sectors of it are singly P-invariant due to the *extended* covariant prescription (75) now demanded by (25). Quite a similar (two-component) scheme primarily applies to both the remaining leptons and the quarks, despite the fact that these are also electrically charged particles. Actually, at the zero-mass stage, the electric charge (just like any other scalar charge) may at most be taken as being defined *only in magnitude*, which is obviously left *unvaried* on passing from the particle to the antiparticle. Let then $|q|^2$ be its square magnitude relevant to a given $(\chi_f, \chi_{\bar{f}})$ field pair, where one has $|q|^2 = 0$ only for χ_f ($\chi_{\bar{f}}$) = χ_ν ($\chi_{\bar{\nu}}$). To express the "absolute strength" (with no relative-sign information) merely specifying such a charge, one can also choose, for convenience, either single conventional root $\mp|q|$ ($\neq 0$) and define a corresponding one-particle charge operator $Q = \mp|q|\, P_{\text{in}}$ with P_{in} acting as in (79). In this way, a "whole" left–right free electric current having an explicit form like (108) can be introduced, where the given sign in front of $|q|$ – the *same* for $\bar{\chi}_f \gamma^\mu \chi_f$ *as* for $\bar{\chi}_{\bar{f}}\, \gamma^\mu \chi_{\bar{f}}$ – just depends on the choice of either $|q|^2$ root.

Let us build from (130) the free Lagrangians

$$\mathcal{L}^{(\text{lepton})} = \mathcal{L}_{\nu\bar{\nu}} + \mathcal{L}_{\ell\bar{\ell}}, \quad \mathcal{L}^{(\text{quark})} = \mathcal{L}_{\text{p}\bar{\text{p}}} + \mathcal{L}_{\text{n}\bar{\text{n}}}, \tag{132}$$

the former being relevant to any given lepton family and the latter to any given quark family. As for $\mathcal{L}^{(\text{lepton})}$, if one is choosing the *minus* sign in front of $|q|$ in (108) – so as to make the $|e|^2$ root *numerically coincident* with the actual electric charge, $-e$, of the final (massive) "charged" lepton – it becomes then appropriate to join together the two fields χ_ν and χ_ℓ into a weak-isospin *doublet*, $D_{\text{L}}^{(\ell)}$, as given by (118). Doing like this, one is automatically selecting the *left-handed* – i.e., $SU(2)_{\text{L}}$ – variety of weak-isospin representations in the leptonic sector. Within such a variety, on the other hand, both antilepton fields $\chi_{\bar{\ell}}$ and $\chi_{\bar{\nu}}$ (covariantly conjugated to χ_ℓ and χ_ν) are to be classified as $SU(2)_{\text{L}}$ *singlets*, just miming the "right-handed lepton" isosinglet ξ_ℓ ($= \chi_{\bar{\ell}}$) and the "right-handed neutrino" isosinglet ξ_ν ($= \chi_{\bar{\nu}}$) of the standard electroweak version. Hence, if $t_{3\text{L}}^{(\text{lepton})}$ generally denotes the eigenvalue of the weak-isospin third component for the case under consideration, we are led to a leptonic Gell-Mann–Nishijima formula consistently reproducing the standard one (with mere *antilepton* isosinglets identically replacing the "right-handed lepton" isosinglets): it looks like

$$-|q| = \left(t_{3\text{L}}^{(\text{lepton})} \, + \, y/2\right) |e|, \tag{133}$$

y being the associated weak-hypercharge eigenvalue, where, of course, $|q| = |e|$ for χ_ℓ *as well as* $\chi_{\bar{\ell}}$, and $|q| = 0$ for χ_ν *as well as* $\chi_{\bar{\nu}}$. This implies that the two fields $\chi_{\bar{\ell}}$ and $\chi_{\bar{\nu}}$, *as far as their belonging to the* $\mathrm{SU}(2)_\mathrm{L}$ *singlet representation is specifically concerned*, are to be assigned the weak-hypercharge eigenvalues $y = -2$ and $y = 0$, respectively. What has just been done for $\mathcal{L}^{(\text{lepton})}$ can likewise be repeated for $\mathcal{L}^{(\text{quark})}$. So, a weak-isospin quark *doublet*, $D_\mathrm{L}^{(\mathrm{q})}$, as given by (119) may be introduced, in such a way that if the (positive) root $+|q| = \frac{2}{3}|e|$ is suitably assigned to both χ_p (p = u, c, t) and $\chi_{\bar{\mathrm{p}}}$, then a (negative) root $-|q| = -\frac{1}{3}|e|$ is automatically left assigned to both χ_n (n = d, s, b) and $\chi_{\bar{\mathrm{n}}}$. A Gell-Mann–Nishijima formula relevant to quarks can be obtained accordingly,

$$\pm|q| = \left(t_{3\mathrm{L}}^{(\text{quark})} + y/2\right)|e|, \tag{134}$$

with mere *antiquark* $\mathrm{SU}(2)_\mathrm{L}$ singlets identically replacing the standard "right-handed quark" isosinglets. One may thus, after all, *naturally* recast both Lagrangians (132) so as to obtain

$$\mathcal{L}^{(\text{lepton})} \to \mathcal{L}^{(\text{lepton})} = \mathcal{L}_{(\mathrm{L})}^{(\text{lepton})}, \quad \mathcal{L}^{(\text{quark})} \to \mathcal{L}^{(\text{quark})} = \mathcal{L}_{(\mathrm{L})}^{(\text{quark})}, \tag{135}$$

where

$$\mathcal{L}_{(\mathrm{L})}^{(\text{lepton})} \equiv \frac{1}{4}\Big[\, i\left(\bar{D}_\mathrm{L}^{(\ell)}\gamma^\mu\partial_\mu D_\mathrm{L}^{(\ell)} + \bar{\chi}_{\bar{\nu}}\,\gamma^\mu\partial_\mu\chi_{\bar{\nu}} + \bar{\chi}_{\bar{\ell}}\,\gamma^\mu\partial_\mu\chi_{\bar{\ell}}\right) + \mathrm{H.c.}\Big] \tag{136}$$

and

$$\mathcal{L}_{(\mathrm{L})}^{(\text{quark})} \equiv \frac{1}{4}\Big[\, i\left(\bar{D}_\mathrm{L}^{(\mathrm{q})}\gamma^\mu\partial_\mu D_\mathrm{L}^{(\mathrm{q})} + \bar{\chi}_{\bar{\mathrm{p}}}\,\gamma^\mu\partial_\mu\chi_{\bar{\mathrm{p}}} + \bar{\chi}_{\bar{\mathrm{n}}}\,\gamma^\mu\partial_\mu\chi_{\bar{\mathrm{n}}}\right) + \mathrm{H.c.}\Big] \tag{137}$$

($\bar{D}_\mathrm{L} = D_\mathrm{L}^\dagger\gamma^0$). These new Lagrangian forms possess manifest global invariance under the group

$$\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y, \tag{138}$$

where $\mathrm{U}(1)_Y$ is generated by a weak-hypercharge variety, Y, *just pertaining* to the $\mathrm{SU}(2)_\mathrm{L}$ representations. Both $\mathcal{L}_{(\mathrm{L})}^{(\text{lepton})}$ and $\mathcal{L}_{(\mathrm{L})}^{(\text{quark})}$ are still invariant under P, as effectively applied via (75), but *neither* of them is any longer invariant under P_in, as defined by (79): actually, as pointed out at the end of the previous section, a *spontaneous* P_in*-violation* occurs on passing from $\mathcal{L}$ to $\mathcal{L}_{(\mathrm{L})}$. This, of course, is a *new* type of "spontaneous symmetry-breaking" (SSB) even though still based on the essential fact that an originally symmetric Lagrangian is rewritten in an asymmetric form. Since parity P $(= P_\mathrm{in}P_\mathrm{ex})$ is left on the whole a symmetry operation, it follows that a P_ex*-violation* should be likewise involved, which, in line with the final remarks of Section 2, can account *by itself* for the actual left–right spatial asymmetry shown up by the weak phenomenology of fermions. As we shall see at the end of the present section, this further kind of SSB can just be related to the choice of some specific covariant "vacuum state" going along only with a *given* root of $|q|^2$. Note, on the other hand, that the symmetry "hidden" by such a choice is a *discrete* one, to which the Goldstone theorem (Goldstone, 1961; Goldstone *et al.*, 1962) does not apply. Of course, each Lagrangian $\mathcal{L}_{(\mathrm{L})}$ above – taken in its own manifest symmetrized form (: $\mathcal{L}_{(\mathrm{L})}$:) – is left invariant not only under P, but also under the ordinary charge-conjugation operation, C, with its matrix representation now properly acting *direct* on chiral fields according to (33). If local invariance under (138) is required as well, this can be fulfilled by adding the two minimal gauge coupling terms

$$\mathcal{L}_{\mathrm{int(L)}}^{(\mathrm{lepton})} = \frac{1}{2}\Big[-g_{\mathrm{L}}\,\bar{D}_{\mathrm{L}}^{(\ell)}\gamma^{\mu}\mathbf{T}_{\mathrm{L}}D_{\mathrm{L}}^{(\ell)}\cdot\mathbf{W}_{\mu} - \frac{1}{2}\,g'_{\mathrm{L}}\big(-\bar{D}_{\mathrm{L}}^{(\ell)}\gamma^{\mu}D_{\mathrm{L}}^{(\ell)} - 2\,\bar{\chi}_{\bar{\ell}}\,\gamma^{\mu}\chi_{\bar{\ell}}\big)B_{\mu}\Big] \tag{139}$$

and

$$\mathcal{L}_{\mathrm{int(L)}}^{(\mathrm{quark})} = \frac{1}{2}\Big[-g_{\mathrm{L}}\,\bar{D}_{\mathrm{L}}^{(\mathrm{q})}\gamma^{\mu}\mathbf{T}_{\mathrm{L}}D_{\mathrm{L}}^{(\mathrm{q})}\cdot\mathbf{W}_{\mu} - \frac{1}{2}\,g'_{\mathrm{L}}\left(\frac{1}{3}\bar{D}_{\mathrm{L}}^{(\mathrm{q})}\gamma^{\mu}D_{\mathrm{L}}^{(\mathrm{q})} + \frac{4}{3}\,\bar{\chi}_{\bar{\mathrm{p}}}\,\gamma^{\mu}\chi_{\bar{\mathrm{p}}} - \frac{2}{3}\,\bar{\chi}_{\bar{\mathrm{n}}}\,\gamma^{\mu}\chi_{\bar{\mathrm{n}}}\right)B_{\mu}\Big], \tag{140}$$

the former to be inserted in (136) and the latter in (137), where g_{L} and g'_{L} amount to the standard coupling constants, and where the Y eigenvalues for each single weak-hypercharge current $\bar{\chi}_f\,\gamma^{\mu}Y\chi_f$ or $\bar{\chi}_{\bar{f}}\,\gamma^{\mu}Y\chi_{\bar{f}}$ are in accordance with Eqs. (133) and (134): in particular, note that there appears no current $\bar{\chi}_{\bar{\nu}}\,\gamma^{\mu}Y\chi_{\bar{\nu}}$ just because $Y\chi_{\bar{\nu}} = 0$. As pointed out in the previous section, the early presence of $\mathrm{SU(2)_L}$-singlet contributions should indeed be connected with the final involvement of (left–right symmetric) currents of the type (108). In either of these $\mathrm{SU(2)_L}\otimes\mathrm{U(1)}_Y$ coupling terms, $\mathbf{T}_{\mathrm{L}}$ is the $\mathrm{SU(2)_L}$ generator defined by (117), and the $\frac{1}{2}$ factor multiplying the whole expression within square brackets is due to the normalized chiral-field definitions (24). The "charged" gauge field $W_{\mu} = 2^{-1/2}(W_{1\mu} - i\,W_{2\mu})$ and its adjoint, $W_{\mu}^{\dagger}$, are such that the former annihilates and the latter creates a (massless) vector boson with a $T_{3\mathrm{L}}$ eigenvalue $t_{3\mathrm{L}} = +1$ and a conventional $|q|^2$ root $+|q| = |e|$ as assigned by either (133) or (134) (recall that only $|q|^2$ makes sense at this stage).

Actually, this is not the only way of proceeding. Starting anew from the fermion–antifermion free Lagrangians (132), one can alternatively make a whole *opposite* choice of $|q|^2$ roots and build both an *antilepton* weak-isospin doublet, $D_{\mathrm{R}}^{(\bar{\ell})}$, as given by (120) and an *antiquark* weak-isospin doublet, $D_{\mathrm{R}}^{(\bar{\mathrm{q}})}$, as given by (121). Doing like this, one is instead selecting the *right-handed* – i.e., $\mathrm{SU(2)_R}$ – variety of the available weak-isospin representations. Within the (equally admissible) new framework so obtained, it is the *fermion* fields that enter as isosinglets, just in the place of the standard "left-handed antifermions." An alternative set of free Lagrangians, dual to those in (132), may thus be built,

$$\mathcal{L}^{(\text{lepton})} \to \mathcal{L}^{(\text{lepton})} = \mathcal{L}^{(\text{lepton})}_{(\text{R})}, \quad \mathcal{L}^{(\text{quark})} \to \mathcal{L}^{(\text{quark})} = \mathcal{L}^{(\text{quark})}_{(\text{R})}, \tag{141}$$

being such that

$$\mathcal{L}^{(\text{lepton})}_{(\text{R})} \equiv \frac{1}{4}\left[i \left(\bar{D}^{(\bar{\ell})}_{\text{R}} \gamma^\mu \partial_\mu D^{(\bar{\ell})}_{\text{R}} + \bar{\chi}_\nu \gamma^\mu \partial_\mu \chi_\nu + \bar{\chi}_\ell \gamma^\mu \partial_\mu \chi_\ell \right) + \text{H.c.}\right] \tag{142}$$

and

$$\mathcal{L}^{(\text{quark})}_{(\text{R})} \equiv \frac{1}{4}\left[i \left(\bar{D}^{(\bar{\text{q}})}_{\text{R}} \gamma^\mu \partial_\mu D^{(\bar{\text{q}})}_{\text{R}} + \bar{\chi}_{\text{p}} \gamma^\mu \partial_\mu \chi_{\text{p}} + \bar{\chi}_{\text{n}} \gamma^\mu \partial_\mu \chi_{\text{n}} \right) + \text{H.c.}\right]. \tag{143}$$

These Lagrangians can just be obtained by applying P_{in} – as defined by (79) – to $\mathcal{L}^{(\text{lepton})}_{(\text{L})}$ and $\mathcal{L}^{(\text{quark})}_{(\text{L})}$, respectively. They possess manifest invariance under the group

$$\text{SU}(2)_{\text{R}} \otimes \text{U}(1)_{\bar{Y}}, \tag{144}$$

where $\text{U}(1)_{\bar{Y}}$ is generated by a weak-hypercharge variety, $\bar{Y}$, *distinct* from Y and *just pertaining* to the $\text{SU}(2)_{\text{R}}$ representations. Likewise the two (no longer suitable) Gell-Mann–Nishijima formulas (133) and (134) should be replaced by the alternative ones

$$+|q| = \left(t^{(\text{lepton})}_{3\text{R}} + \bar{y}/2\right)|e| \tag{145}$$

and

$$\mp|q| = \left(t^{(\text{quark})}_{3\text{R}} + \bar{y}/2\right)|e|, \tag{146}$$

where, for instance, it is the root $+|q| = |e|$ that has now been assigned to both χ_ℓ and $\chi_{\bar{\ell}}$. The additional requirement of local invariance under (144) clearly leads to the introduction of a minimal coupling term

$$\mathcal{L}^{(\text{lepton})}_{\text{int(R)}} = \frac{1}{2}\left[-g_{\text{R}}\, \bar{D}^{(\bar{\ell})}_{\text{R}} \gamma^\mu \mathbf{T}_{\text{R}} D^{(\bar{\ell})}_{\text{R}} \cdot \mathbf{W}_\mu - \frac{1}{2}\, g'_{\text{R}} \left(\bar{D}^{(\bar{\ell})}_{\text{R}} \gamma^\mu D^{(\bar{\ell})}_{\text{R}} + 2\, \bar{\chi}_\ell\, \gamma^\mu \chi_\ell \right) B_\mu \right], \tag{147}$$

to be inserted in (142), and a minimal coupling term

$$\mathcal{L}^{(\text{quark})}_{\text{int(R)}} = \frac{1}{2}\Bigg[-g_{\text{R}}\, \bar{D}^{(\bar{\text{q}})}_{\text{R}} \gamma^\mu \mathbf{T}_{\text{R}} D^{(\bar{\text{q}})}_{\text{R}} \cdot \mathbf{W}_\mu - \frac{1}{2}\, g'_{\text{R}} \left(-\frac{1}{3} \bar{D}^{(\bar{\text{q}})}_{\text{R}} \gamma^\mu D^{(\bar{\text{q}})}_{\text{R}} - \frac{4}{3}\, \bar{\chi}_{\text{p}}\, \gamma^\mu \chi_{\text{p}} + \frac{2}{3}\, \bar{\chi}_{\text{n}}\, \gamma^\mu \chi_{\text{n}} \right) B_\mu \Bigg], \tag{148}$$

to be inserted in (143), $\mathbf{T}_{\text{R}}$ being the $\text{SU}(2)_{\text{R}}$ generator defined by (117). In this *alternative* (but equivalent) covariant dynamical description, one has that $-\mathbf{T}$, rather than $\mathbf{T}$, is the *effective* SU(2) generator resulting from the application of $\mathbf{T}_{\text{R}}$ to D_{R}. So, differently from the $\text{SU}(2)_{\text{L}} \otimes \text{U}(1)_Y$ approach, the two "charged" gauge fields W_μ and $W_\mu^\dagger$ should now be understood (the former) to annihilate and (the latter) to create a (massless) vector boson with a T_{3R} eigenvalue $t_{3R} = -1$ and a corresponding $|q|^2$ root $-|q| = -|e|$ as assigned according to (145) or (146).

It is worth spending some more words concerning the weak hypercharge, which may here appear in two *distinct* varieties, Y and $\bar{Y}$, depending on whether the $\mathrm{SU}(2)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ or $\mathrm{SU}(2)_\mathrm{R}\otimes\mathrm{U}(1)_{\bar{Y}}$ covariant description is being adopted. One is allowed to put

$$Y = Y^{(1/2)}(-\gamma^5)X_\mathrm{L} + Y^{(0)}(-\gamma^5)X_\mathrm{R}\ ,\ \ \bar{Y} = Y^{(1/2)}(-\gamma^5)X_\mathrm{R} + Y^{(0)}(-\gamma^5)X_\mathrm{L}, \tag{149}$$

where $Y^{(1/2)}$ and $Y^{(0)}$ are suitable one-particle scalar operators acting in the isodoublet and isosinglet ordinary spaces, respectively, and where X_L and X_R are the pair of chiral-projection operators defined in (117). Hence it follows, in covariant terms, either that

$$YD_\mathrm{L} = Y^{(1/2)}(-\gamma^5)D_\mathrm{L}, \quad Y\chi_{\bar{f}} = Y^{(0)}(-\gamma^5)\chi_{\bar{f}} \tag{150}$$

or that

$$\bar{Y}D_\mathrm{R} = Y^{(1/2)}(-\gamma^5)D_\mathrm{R}, \quad \bar{Y}\chi_f = Y^{(0)}(-\gamma^5)\chi_f \tag{151}$$

according to the description chosen, where the shared quantities $Y^{(1/2)}(-\gamma^5)$ and $Y^{(0)}(-\gamma^5)$ give evidence of the actual *pseudoscalar* nature of the weak hypercharge in either description. An "overall" weak-hypercharge covariant operator can correspondingly be built by taking the sum of Y and $\bar{Y}$:

$$Y + \bar{Y} = \left[Y^{(1/2)} + Y^{(0)}\right](-\gamma^5). \tag{152}$$

Both alternative covariant coupling pairs (139), (140) and (147), (148) – coming from two pairs of P-*invariant* free Lagrangians – can themselves be left P-invariant if the gauge fields $\mathbf{W}_\mu$ and B_μ are simply assumed to be axial-vectors in space–time. This is because the (left- and right-handed) weak-isospin currents, as covariantly rewritten – with the help of Eqs. (115) and (117) – in terms of $\mathbf{T}^\mathrm{w}$,

$$\bar{D}_\mathrm{L}\gamma^\mu\mathbf{T}_\mathrm{L}D_\mathrm{L} = \bar{D}_\mathrm{L}\gamma^\mu\mathbf{T}^\mathrm{w}D_\mathrm{L}, \quad \bar{D}_\mathrm{R}\gamma^\mu\mathbf{T}_\mathrm{R}D_\mathrm{R} = \bar{D}_\mathrm{R}\gamma^\mu\mathbf{T}^\mathrm{w}D_\mathrm{R}, \tag{153}$$

do already behave as axial-vectors under

$$D_\mathrm{L,R} \longrightarrow \gamma^0 D_\mathrm{L,R}, \quad \bar{D}_\mathrm{L,R} \longrightarrow \bar{D}_\mathrm{L,R}\gamma^0\,; \tag{154}$$

and the same clearly holds for the associated weak-hypercharge currents, in view of (150) and (151). Note, on the other hand, that neither Eqs. (133), (134) nor Eqs. (145), (146) may strictly have P-invariant forms, since each single root $\mp|q|$ is by itself a scalar (rather than a pseudoscalar) quantity. In the light of what has been previously argued, such an "asymmetry" is however of no physical relevance and does not imply any real P-breaking: as long as there are mere *chiral fields* involved, only $|q|^2$ can make sense, and the sign of either $|q|^2$ root can be nothing but a matter of convention. Thus, on applying P (P_{in}) we may also take the liberty of (conventionally) *inverting* all $|q|^2$ roots (including those associated with the gauge bosons). Due to (114), doing like this is the same as applying $C_{\text{cov}}P$ ($C_{\text{cov}}P_{\text{in}}$) instead of P (P_{in}); which makes indeed no difference, owing to the exclusive presence of C_{cov}-invariant (chiral-field) currents, coupled accordingly to C_{cov}-invariant gauge fields. On this understanding, whether one is concerned with leptons or quarks, the reversal $\pm|q| \to \mp|q|$ may well be said to be *an effect implicitly induced by* P_{in} ($= C_{\text{cov}}P_{\text{in}}$) *as a result of the transformation*

$$P_{\text{in}}: \; (t_{3\text{L}} + y/2)\,|e| \longrightarrow (t_{3\text{R}} + \bar{y}/2)\,|e|. \tag{155}$$

All the couplings above can also be naturally assumed to be invariant under the (ordinary) charge conjugation C, as properly applied to chiral-field currents in the way (33) (which just enables the usual C-matrix to induce *chirality inversion* as well). Concerning the charged-current coupling terms, the net effect produced by C will be to turn them into their own Hermitian conjugates. As further regards the neutral-current coupling terms, their normally ordered versions will behave under C in line with (107), so that C invariance may simply be guaranteed by setting

$$C^\dagger\, W_{3\mu}\, C = W_{3\mu}, \quad C^\dagger\, B_\mu\, C = B_\mu. \tag{156}$$

Yet, in accordance with what happens for the two original, P_{in} *mirror* free-Lagrangian pairs (136), (137) and (142), (143), it can easily be seen that neither (139), (140) nor (147), (148) are invariant under P_{in}. For instance, starting from (139) and recalling both (104) and (70), we obtain

$$P_{\text{in}}: \begin{cases} \bar{D}^{(\ell)}_{\text{L}}\gamma^\mu \mathbf{T}^{\text{w}} D^{(\ell)}_{\text{L}} \longrightarrow \bar{D}^{(\bar{\ell})}_{\text{R}}\gamma^\mu \mathbf{T}^{\text{w}} D^{(\bar{\ell})}_{\text{R}} \\ \big(-\bar{D}^{(\ell)}_{\text{L}}\gamma^\mu D^{(\ell)}_{\text{L}} - 2\,\bar{\chi}_{\bar{\ell}}\,\gamma^\mu \chi_{\bar{\ell}}\big) \longrightarrow \big(\bar{D}^{(\bar{\ell})}_{\text{R}}\gamma^\mu D^{(\bar{\ell})}_{\text{R}} + 2\,\bar{\chi}_{\ell}\gamma^\mu \chi_{\ell}\big)\,; \end{cases} \tag{157}$$

and hence it also follows that along with (124), one consistently has

$$P_{\text{in}}: \text{U}(1)_Y \rightleftharpoons \text{U}(1)_{\bar{Y}}. \tag{158}$$

So, putting

$$g_{\text{L}} = g_{\text{R}} \equiv g, \quad g'_{\text{L}} = g'_{\text{R}} \equiv g' \tag{159}$$

as just required by left–right equivalence, it can likewise be concluded that Eqs. (147), (148) are the P_{in} *mirror counterparts* of Eqs. (139), (140). Of course, such a correspondence holds provided that P_{in} itself (besides C_{cov}) is assumed to leave all the gauge fields formally unvaried. As in particular regards W_μ and $W^\dagger_\mu$, recall, however, that on passing from the $\text{SU}(2)_\text{L}\otimes\text{U}(1)_Y$ to the $\text{SU}(2)_\text{R}\otimes\text{U}(1)_{\bar{Y}}$ picture, an *interchange* necessarily occurs between the physical roles played by them; so that they should more properly be said to turn into

$$W_\mu = W_\mu^{(P_{\rm in})\dagger}\ , \quad W_\mu^\dagger = W_\mu^{(P_{\rm in})}. \tag{160}$$

All that can be conveniently recast by means of a *unified* formalism joining together both of the above (equally available) $SU(2)_L \otimes U(1)_Y$ and $SU(2)_R \otimes U(1)_{\bar{Y}}$ covariant approaches. These originate in the two (equally admissible) alternative positions (135) and (141); and either of them is implicitly related to a given choice of $|q|^2$ roots, which is *opposite* to that for the other approach. We thus have that the two sets of $SU(2)_L \otimes U(1)_Y$ and $SU(2)_R \otimes U(1)_{\bar{Y}}$ representations may just be "labelled" according to such choices. More precisely, each single starting free Lagrangian (132) may be given, by use of both (135) and (141), a "whole" manifest $L + R$ form like

$$\mathcal{L} = \mathcal{L}_{(L)}\,\mathcal{P}_+ \,+\, \mathcal{L}_{(R)}\,\mathcal{P}_-, \tag{161}$$

where $\mathcal{P}_+$ and $\mathcal{P}_-$ are indeed two suitable *Casimir operators*, such that

$$\begin{aligned} &\mathcal{P}_+ \equiv |+\rangle\langle+|, \qquad \mathcal{P}_- \equiv |-\rangle\langle-|\ , \\ &\mathcal{P}_+ + \mathcal{P}_- = 1, \qquad \mathcal{P}_+\mathcal{P}_- = \mathcal{P}_-\mathcal{P}_+ = 0. \end{aligned} \tag{162}$$

In view of the implicit effect (114) induced by the $P_{\rm in}$ $(= C_{\rm cov} P_{\rm in})$ operation, the explicit presence of either $\mathcal{P}_+$ (coupled to $\mathcal{L}_{(L)}$) or $\mathcal{P}_-$ (coupled to $\mathcal{L}_{(R)}$) implies a *spontaneous* $C_{\rm cov}$*-symmetry-breaking*, which corresponds to the introduction of two, no longer $C_{\rm cov}$-symmetric, vacuum states,

$$|0\rangle|+\rangle, \qquad |0\rangle|-\rangle, \tag{163}$$

just requiring that the symmetry transformation (80) be supplemented by the "vacuum asymmetry" effects

$$C_{\rm cov}^\dagger\,\mathcal{P}_+\,C_{\rm cov} = \mathcal{P}_-, \quad C_{\rm cov}^\dagger\,\mathcal{P}_-\,C_{\rm cov} = \mathcal{P}_+. \tag{164}$$

The explicit presence in question, on the other hand, does not affect the spontaneous breaking of $P_{\rm in}$ symmetry already included in either (135) or (141), so that the spontaneously-broken transformation (79) should merely be supplemented by

$$P_{\rm in}^\dagger\,\mathcal{P}_+\,P_{\rm in} = \mathcal{P}_+, \quad P_{\rm in}^\dagger\,\mathcal{P}_-\,P_{\rm in} = \mathcal{P}_-. \tag{165}$$

Similarly, the "whole" weak hypercharge $Y + \bar{Y}$, as given by (152), may read

$$Y + \bar{Y} = \left[Y^{(1/2)} + Y^{(0)}\right](-\gamma^5)(\mathcal{P}_+ + \mathcal{P}_-), \tag{166}$$

with

$$Y = \left[Y^{(1/2)} + Y^{(0)}\right](-\gamma^5)\,\mathcal{P}_+, \quad \bar{Y} = \left[Y^{(1/2)} + Y^{(0)}\right](-\gamma^5)\,\mathcal{P}_- \tag{167}$$

just ensuring that

$$Y\,|-\rangle = \bar{Y}\,|+\rangle = 0. \tag{168}$$

In this way, one is actually dealing with a *unified* Fock space, $\mathcal{F}^{(+)} \oplus \mathcal{F}^{(-)}$, that includes two general covariant Fock-space varieties, $\mathcal{F}^{(+)}$ and $\mathcal{F}^{(-)}$, pertaining to the $\mathrm{SU(2)_L}\otimes\mathrm{U(1)}_Y$ and the $\mathrm{SU(2)_R}\otimes\mathrm{U(1)}_{\bar{Y}}$ picture, respectively, and being distinguished by the two "dressed" vacuum states (163). Thanks to (168), one is also allowed to say that the form (161) has manifest global invariance under the *left-right symmetric* gauge group

$$\begin{aligned}[\,\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y] &\oplus [\mathrm{SU(2)_R} \otimes \mathrm{U(1)}_{\bar{Y}}\,] \\ &= [\,\mathrm{SU(2)_L} \oplus \mathrm{SU(2)_R}] \otimes [\mathrm{U(1)}_Y \oplus \mathrm{U(1)}_{\bar{Y}}\,]\,, \end{aligned} \tag{169}$$

where $\mathrm{SU(2)_L} \oplus \mathrm{SU(2)_R}$ is generated by (115), and $\mathrm{U(1)}_Y \oplus \mathrm{U(1)}_{\bar{Y}}$ by (166). Quite an analogous procedure can be exploited to join together the pair of equivalent minimal gauge couplings in turn ensuring $\mathrm{SU(2)_L}\otimes\mathrm{U(1)}_Y$ and $\mathrm{SU(2)_R}\otimes\mathrm{U(1)}_{\bar{Y}}$ local invariance; thus, a "whole" L + R gauge coupling of the general type

$$\mathcal{L}_{\mathrm{int(L+R)}} = \mathcal{L}_{\mathrm{int(L)}}\mathcal{P}_+ \,+\, \mathcal{L}_{\mathrm{int(R)}}\mathcal{P}_-, \tag{170}$$

can be built, which is suitable also to ensure local invariance under the overall group (169) (with a total number of *two* L + R coupling constants, g and g', correspondingly involved). Of course, each original free Lagrangian (130) shows a C_{cov}-symmetric (besides P_{in}-symmetric) form, if taken simply as $\mathcal{L} = \mathcal{L}\,\mathcal{P}_+ + \mathcal{L}\,\mathcal{P}_-$: the spontaneous breakings of C_{cov} and P_{in} symmetries are together introduced just on setting

$$\mathcal{L}\,\mathcal{P}_+ + \mathcal{L}\,\mathcal{P}_- = \mathcal{L}_{\mathrm{(L)}}\,\mathcal{P}_+ \,+\, \mathcal{L}_{\mathrm{(R)}}\,\mathcal{P}_- \tag{171}$$

and on associating the $\mathrm{SU(2)_L}\otimes\mathrm{U(1)}_Y$-symmetric Lagrangian form $\mathcal{L} = \mathcal{L}_{\mathrm{(L)}}$ with the special vacuum $|0\rangle|+\rangle$, and the $\mathrm{SU(2)_R}\otimes\mathrm{U(1)}_{\bar{Y}}$-symmetric Lagrangian form $\mathcal{L} = \mathcal{L}_{\mathrm{(R)}}$ with the special vacuum $|0\rangle\,|-\rangle$. Note, however, that both (161) and (170) turn out still to be invariant under the overall operation $C_{\mathrm{cov}}\,P_{\mathrm{in}}$, whose effect is obtained by combining (164) with

$$P_{\mathrm{in}} : \mathcal{L}_{\mathrm{(L)}} \rightleftharpoons \mathcal{L}_{\mathrm{(R)}}, \quad \mathcal{L}_{\mathrm{int(L)}} \rightleftharpoons \mathcal{L}_{\mathrm{int(R)}}. \tag{172}$$

7. MASS APPEARANCE AS A NECESSARY CONDITION FOR A SCALAR CHARGE TO YIELD A LOCAL GAUGE COUPLING

A peculiar internal feature of the "dressed" fermion quantum field formalism developed in Secs. 2–4 lies in the predicted coexistence of two *anticommuting*, scalar and pseudoscalar, varieties of charges. Due to it, the activation of some *superselective* mechanism is now generally required, in order that a charge may give rise to a local gauge coupling. In the case of a massive fermion, such a mechanism is switched on automatically, by the explicit involvement of a *one-particle operator* representing the charge (and being defined in the "dressing" fermion–antifermion covariant internal space S_{in}): for instance, if Q is the one-particle operator associated with a given scalar charge, then applying Q (from the right) to the unified covariant fermion–antifermion field $\Psi(x)$ has just the result of *superselecting* the "Dirac" S_{in} representation of $\Psi(x)$ (i.e., the one in which Q itself is made diagonal). On passing to the zero-mass case, the nonstandard feature above is brought to its extreme consequences: these are a permanent superselection rule for pseudoscalar charges, on one hand, and a permanent *anti*-superselection rule for scalar charges, on the other. Under such special constraints, the following statement can be shown to hold:

Theorem 1. *In principle, any scalar charge carried by a massless spin-$\frac{1}{2}$ fermion (antifermion) should be strictly prevented from generating a local gauge coupling; such a goal could be realized only if the fermion (antifermion) is simultaneously induced to acquire a mass.*

The proof is based just on the new, pure and simple *specular* internal models universally applying to massless spin-$\frac{1}{2}$ point fermions and related antifermions. Suppose the actual existence in Nature of two such particles carrying at least one nonzero scalar charge. By use of the P-invariant Lagrangian (73), a unified, fully covariant description of *them both* is now available, in terms of two fields, χ_f and $\chi_{\bar{f}}$, having definite and opposite chiralities. These, however, are the pure *chirality conjugates* of each other and cannot stand at all for two (mutually conjugate) "scalar-charge eigenfields." The fermion and the antifermion so conjectured would therefore behave like particles with a scalar charge being subject to a *maximal uncertainty* in sign. Hence, the exact *opposite* of a superselection rule should hold for that charge – whose "eigenfields" might be thought of only as *fifty-fifty mixtures* of χ_f and $\chi_{\bar{f}}$ – and there could be *no* mechanism diagonalizing it (and so allowing it to generate a local gauge coupling). One clearly has, on the other hand, that the appearance of a *mass* for such a pair of particles would both remove the anti-superselection rule in question and allow an actual diagonalization of the scalar charge.

The theorem above, if applied to the electroweak scheme, sets *anew* the question about spontaneous symmetry-breaking (SSB) and fermion masses. According to the standard approach, there seems to be no reason (inherent in electroweak dynamics) for the appearance of fermion masses: the Higgs couplings to fermions are just inserted *ad hoc*, so that theory may fit in with experience. On passing to the present approach, the viewpoint is radically changed; and the fact that every pointlike fermion with at least one scalar charge is also a *massive* particle can no longer be taken as occurring by chance. Herein, on the contrary, the appearance of fermion masses (via SSB) is strictly made an *essential internal requirement* of the model: without it, no actual scalar-charge eigenstates (with nonzero eigenvalues) could ever be obtained, and no scalar charges (such as the electric and color ones) would ever be able to generate local gauge couplings. Such a viewpoint should more properly demand some SSB mechanism no longer having a presumed "external" origin (connected with the existence of the Higgs particle). This is indeed reinforced by the following new basic theoretical result directly proceeding from the model of Section 4: *even a massless spin-$\frac{1}{2}$ fermion, and not only a massless spin-*1 *boson, needs to gain an extra helicity freedom degree for it to be made massive*. If so, then the generation of a fermion mass should likewise be expected to be rather obtained from the "absorption" of a suitable would-be-Goldstone boson (than from the coupling to a new, yet undiscovered, real particle).

8. A UNIFIED, PURE "INTERNAL" MECHANISM OF MASS GENERATION, WITH AN EXTRA WOULD-BE-GOLDSTONE BOSON REPLACING THE HIGGS PARTICLE

Take, for a moment, the Higgs field doublet $\phi(x)$ as usually parametrized in terms of the four Hermitian scalar fields $\theta_1(x)$, $\theta_2(x)$, $\theta_3(x)$, and $\eta(x)$, where the $\theta_i(x)$'s ($i = 1, 2, 3$) stand for the three (massless) would-be-Goldstone bosons, and $\eta(x)$ stands for the (massive) Higgs boson:

$$\phi(x) = \exp\left[i \sum_{k=1}^{3} \theta_k(x) \frac{\tau_k}{2} \right] \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v + \eta(x) \end{pmatrix} \tag{173}$$

(v being a real constant). As is well-known, the $\phi(x)$ Lagrangian is of the type

$$\mathcal{L}_{\text{Higgs}} = (\partial^\mu \phi)^\dagger \partial_\mu \phi - V(\phi^\dagger \phi) \tag{174}$$

with

$$V(\phi^\dagger \phi) = a\, \phi^\dagger \phi + b\, (\phi^\dagger \phi)^2, \qquad (a < 0,\ b > 0). \tag{175}$$

Recently, it has been pointed out (Ziino, 2003) that in the standard electroweak framework, such a manner of parametrizing $\phi(x)$ – so as to make it reducible to a pure *Hermitian* field by a suitable SU(2) transformation – is not fully consistent with the fact that $\phi(x)$ is also expected to bear a nonzero "charge" (i.e., the weak hypercharge) which is further a *scalar* relative to rotations in weak-isospin space: strictly speaking, one should have rather to do with a field $\phi(x)$ that would be left accordingly a *non-Hermitian* (i.e., "charged") field under *whatever* SU(2) gauge transformation, including the one defining the "unitary gauge" itself. It has been shown as well that the only possible way out of such an impasse is to re-parametrize

$\phi(x)$ in the new, SU(2)⊗U(1)-covariant form

$$\phi(x) = \exp\left[i \sum_{k=1}^{3} \theta_k(x) \frac{\tau_k}{2}\right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \exp[i\eta(x)] \end{pmatrix}$$

$$= \exp\left\{i \left[\sum_{k=1}^{3} \theta_k(x) \frac{\tau_k}{2} + \eta(x)\right]\right\} \phi_\circ \tag{176}$$

where

$$\phi_\circ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{177}$$

is the (real) vacuum expectation value of $\phi(x)$, and where $\eta(x)$ now stands for a *fourth* (massless) would-be-Goldstone boson (replacing the Higgs particle). At first sight, this would seem to violate the constraint of no more than *three* would-be-Goldstone bosons in all (which follows from the invariance property of $\phi_\circ$ under the electromagnetic gauge subgroup) (Cheng and Li, 1984). Such a constraint, however, holds *only if* $V(\phi^\dagger\phi) \neq constant$. As can be checked by substituting (176) into (175), we now have, on the contrary,

$$V(\phi^\dagger\phi) = a\,\frac{v^2}{2} + b\,\frac{v^4}{4} = \text{constant}; \tag{178}$$

so that we may consistently obtain a mass matrix

$$(M^2)_{ik} = \frac{\partial^2 V}{\partial\phi_i\,\partial\phi_k} = 0, \qquad (i, k = 1, 2, 3, 4) \tag{179}$$

for all *four* Hermitian fields ϕ_i generally chosen to parametrize $\phi(x)$. As the constant term (178) may clearly be taken away from (174), the new parametrization (176) also allows us to reduce (174) to a pure *free* Lagrangian,

$$\mathcal{L}_{\text{Higgs}} = (\partial^\mu\phi)^\dagger\,\partial_\mu\phi, \tag{180}$$

with ϕ *always keeping*, however, its own vacuum expectation value (177). Of course, the (massless) field $\eta(x)$ appearing in (176) can be eliminated by carrying out the U(1) gauge transformation

$$\phi(x) \to \exp[-i\eta(x)]\,\phi(x). \tag{181}$$

An *extended*, SU(2)⊗U(1)-covariant, unitary gauge should be introduced accordingly, which may permit the simultaneous elimination of $\eta(x)$ as well as of $\theta_1(x)$, $\theta_2(x)$, and $\theta_3(x)$: it is defined by

$$\phi^{\text{ex}}(x) \equiv \exp\left\{-i \left[\sum_{k=1}^{3} \theta_k(x) \frac{\tau_k}{2} + \eta(x)\right]\right\} \phi(x) = \phi_\circ. \tag{182}$$

This can be more appropriately re-expressed in the form

$$\phi^{\mathrm{ex}}(x) = U \exp[-i\, Y_{\mathrm{st}}\eta(x)]\,\phi(x) \tag{183}$$

where U is the SU(2) transformation characterizing the standard unitary gauge,

$$U \equiv \exp\left[-i\sum_{k=1}^{3}\theta_k(x)\frac{\tau_k}{2}\right], \tag{184}$$

and Y_{st} is a one-particle standard operator of weak hypercharge, being such that $Y_{\mathrm{st}}\phi(x) = (+1)\,\phi(x)$. As for the way how the original SU(2)⊗U(1) gauge fields, $\mathbf{W}_\mu(x)$ and $B_\mu(x)$, transform on passing to the new unitary gauge, the only novelty is that $B_\mu(x)$ is no longer left invariant: writing the covariant derivative for $\phi(x)$ as $D_\mu = \partial_\mu + ig\mathbf{T}\cdot\mathbf{W}_\mu + i(g'/2)B_\mu(x)$, with $\mathbf{T} = (\frac{1}{2}\tau_1, \frac{1}{2}\tau_2, \frac{1}{2}\tau_3)$, and denoting the transformed fields by $\mathbf{W}^{\mathrm{ex}}_\mu(x)$ and $B^{\mathrm{ex}}_\mu(x)$, one has

$$\begin{aligned}\mathbf{T}\cdot\mathbf{W}^{\mathrm{ex}}_\mu(x) &= U\mathbf{T}\cdot\mathbf{W}_\mu(x)U^{-1} - i\,(1/g)\,U\partial_\mu U^{-1}\\ B^{\mathrm{ex}}_\mu(x) &= B_\mu(x) + (2/g')\,\partial_\mu\eta(x)\end{aligned} \tag{185}$$

where U is given by (184). This, however, does not imply any physical changes, as one always obtains (in terms of the ordinary Weinberg mixing angle θ_W) the two mass eigenfields

$$\begin{cases} Z_\mu = \cos\theta_W\, W^{\mathrm{ex}}_{3\mu} - \sin\theta_W\, B^{\mathrm{ex}}_\mu \\ A_\mu = \sin\theta_W\, W^{\mathrm{ex}}_{3\mu} + \cos\theta_W\, B^{\mathrm{ex}}_\mu \end{cases} \tag{186}$$

the former having the usual mass $M_Z = gv/2\cos\theta_W$ and the latter being massless. Actually, the inclusion of the factor $\exp[-i\eta(x)]$ in the definition of the unitary gauge appears to be immaterial to get the IVB masses, and the mere standard unitary gauge is still sufficient for such a purpose. One has, on the contrary, that the fermions are now strictly *massless* in the standard unitary gauge: they can be made massive *only if* $\eta(x)$ *is eliminated as well* (by the more appropriate choice of the *extended* unitary gauge). On the other hand, the removal of $\eta(x)$ clearly implies that the associated freedom degree should somehow reappear *elsewhere*. This indeed corresponds to the fact that any weakly-interacting original fermion happens to gain one *extra* helicity freedom degree on acquiring a mass: the phenomenological primary constraint of only one (negative) helicity eigenvalue turns out to be removed, and also a positive-helicity eigenstate is made allowable for such a fermion. We are thus led, after all, to assert that *a Higgs mechanism of disappearance of* $\eta(x)$ *would seem just to be essential for the original* (only left-handed) *weakly-interacting quarks and charged leptons to be made massive*.

Another fundamental question to deal with is the one concerning the renormalizability and unitarity of the theory, which must survive *despite the absence of the Higgs-particle contributions*. Actually, in view of what stated above, we now have that *leptons and quarks are left massless in any gauges where the extra would-be-Goldstone freedom degree* $\eta(x)$ *is still present*. This feature may be expected to be of crucial importance on the basis of the following fact: in the standard approach to electroweak processes, the Higgs cancellations of high-energy unitarity violations generally require Higgs couplings (to fermions and gauge bosons) proportional to the fermion and the gauge-boson masses themselves, which just implies that for *vanishing* masses, such violations would disappear *automatically* (i.e., with no Higgs couplings). The problem is left, however, of strictly defining a class of *extended* renormalizable (R) gauges being able both to generalize 't Hooft's gauges and to remedy the apparent failure of unitarity due to the absence of the Higgs particle. For this purpose, it is appropriate to set

$$\rho(x) = \exp[i\eta(x)] \equiv \rho^{(+)}(x), \quad \rho^{\dagger}(x) = \exp[-i\eta(x)] \equiv \rho^{(-)}(x), \tag{187}$$

$\rho^{\dagger}$ belonging to the charge-conjugate isodoublet $i\tau_2\widetilde{\phi}^{\dagger}$ (with weak hypercharge -1). A general "Yukawa" fermion-coupling term may also be introduced: this, in the standard (and not extended) unitary gauge, reads

$$\mathcal{L}^{(\mathrm{f})}_{\mathrm{Yukawa}} = \frac{1}{\sqrt{2}} v \sum_{i,j} f_{ij} \bar{\psi}^{(\mp)}_{i\mathrm{L}} \rho^{(\pm)} \psi_{j\mathrm{R}} + \mathrm{H.c.} \tag{188}$$

where $\bar{\psi}_{i\mathrm{L}} = \bar{\psi}_i(1+\gamma^5)\frac{1}{2}$ and $\psi_{j\mathrm{R}} = \frac{1}{2}(1+\gamma^5)\psi_j$, and where the f_{ij}'s are suitable coupling constants depending on the fermion flavor indices i, j. The superscript $(\mp)$ attached to $\bar{\psi}_{i\mathrm{L}}$ refers to either sign of the associated weak-isospin third component (according to whether the fermion isodoublet is coupled to ϕ or $i\tau_2\widetilde{\phi}^{\dagger}$, respectively). In (188), due to the presence of $\eta(x)$, both $\bar{\psi}_{i\mathrm{L}}$ and $\psi_{j\mathrm{R}}$ fermion fields are strictly *massless*, and $\mathcal{L}^{(\mathrm{f})}_{\mathrm{Yukawa}}$ itself is a *pure* interaction Lagrangian. Note that in this same gauge, on the contrary, one usually gets an interaction (Higgs) term *plus* a mass term. On passing to the *extended* unitary gauge, $\eta(x)$ is eliminated and $\mathcal{L}^{(\mathrm{f})}_{\mathrm{Yukawa}}$ may take the wanted form

$$\mathcal{L}^{(\mathrm{f})}_{\mathrm{Yukawa}} = \mathcal{L}^{(\mathrm{f})}_{\mathrm{mass}} \equiv \sum_{i,j} m_{ij} \bar{\psi}^{\mathrm{ex}}_{iL} \psi^{\mathrm{ex}}_{jR} + \mathrm{H.c.} \tag{189}$$

with $m_{ij} = \frac{1}{\sqrt{2}} v f_{ij}$ and

$$\bar{\psi}^{\mathrm{ex}}_{iL} = \bar{\psi}_{i\mathrm{L}} \exp[i\, Y_{\mathrm{st}}\eta(x)]\ , \ \psi^{\mathrm{ex}}_{jR} = \exp[-i\, Y_{\mathrm{st}}\eta(x)]\, \psi_{j\mathrm{R}}, \tag{190}$$

Y_{st} being the one-particle (standard) operator of weak hypercharge. In such a gauge, of course, it is at the expense of the $\eta(x)$ would-be-Goldstone boson that the original (pure left-handed) weakly-interacting charged fermions have been made massive and have gained an extra helicity freedom degree. Of course, it may conversely be argued that Eq. (188) stands just for Eq. (189) as recast in an *extended* new gauge still including the standard unitary one. The single would-be-Goldstone boson field $\eta(x)$ peculiar to this gauge is associated with a (standard) propagator $1/k^2$ (k^2 being the square magnitude of the four-momentum carried by η). The corresponding propagator for $\rho(x)$ is not so immediate to find. Yet, we may choose an *intermediate* ("primed") gauge, such that

$$\eta \longrightarrow \eta' = \eta'(\epsilon) = (1-\epsilon)\eta, \qquad (0 < \epsilon < 1). \tag{191}$$

In this alternative gauge, the free field equation $\partial^\mu \partial_\mu \eta = 0$ is turned into

$$(1-\epsilon)^{-1}\partial^\mu \partial_\mu \eta' = 0 \tag{192}$$

and the propagator for the transformed field η' correspondingly reads

$$\frac{(1-\epsilon)}{k^2}. \tag{193}$$

The extended unitary gauge (in which also η is absent) can clearly be approached, in a *continuous* manner, by taking the limit $\epsilon \to 1$. Suppose then $(1-\epsilon)$ to be an *infinitesimal*, and consider the special "primed" gauge thus defined. By means of such a choice, the relations between $\rho'^{(\pm)} = \exp[\pm i\eta']$ and η' are made *linear*, and each actual coupling to $\rho'^{(\pm)}$ – which (due to the survival of the η freedom degree) still involves true *massless* spin-$\frac{1}{2}$ fields – can simplier be handled in terms of an *effective* (seemingly massive-field) coupling to η' only, with a corresponding propagator (193): just as for the general case of a whole power series expansion of $\rho'^{(\pm)}$, what is looking like a "mass" sector in the coupling to $\rho'^{(\pm)}(\epsilon \to 1)$ may at most be regarded as a "would-be-mass" sector, since (with η being still present) it does not yet correspond to a *real* gain of the further helicity freedom degree needed by every primary fermion to acquire a mass. In a gauge like this, *infinitely close* to (but never strictly coincident with) the extended unitary one, the limiting condition of manifest unitarity can still be obtained both for leptons and for quarks (despite the absence of the Higgs particle) *thanks to their masslessness*. The same may apply also to the intermediate-vector-boson (IVB) dynamics, provided that the standard unitary gauge (still included in the considered new gauge) is likewise replaced by a 't Hooft gauge which is *infinitely close* to it (and in which all gauge bosons are left *massless*): this, on the other hand, is nothing but the usual ('t Hooft) procedure for *correctly approaching* the propagator of a massive spin-1 boson. The whole special gauge so obtained can be said to belong to a class of *extended* 't Hooft gauges. Thus (owing to gauge invariance) the unitarity of the theory appears to be fully ensured, and the renormalizability of it is made manifest by the actual existence of a suitable class of *extended* R gauges (just generalizing 't Hooft ones) with good high-energy behaviors of all the relevant propagators, *including* (193).

To sum up, it has been seen that looking at the standard manner of parametrizing the Higgs doublet $\phi(x)$, one could already object that the isoscalar nature of the (nonzero) weak hypercharge carried by $\phi(x)$ would rather demand $\phi(x)$ to be so re-parametrized as to be consistently left a non-Hermitian field under *any*

SU(2) gauge transformation (with no exception even for the special one defining the standard unitary gauge). The rigorous fulfilment of such a condition has been shown to imply a modified mass-generating mechanism, which should be merely based on the gauge elimination of would-be-Goldstone bosons (now *four* in number) and should no longer require the existence of an additional (massive) real particle like the Higgs boson.

A strict "internal" mechanism like this – with a mere "vacuum" Higgs field involved – seems to be particularly appropriate for the whole new electroweak formulation here proposed, within which it can further acquire fully sound theoretical motivations. Indeed, according to the two-component model of Section 4, the primary weakly-interacting massless fermions are to be left-handed *by their own nature*, so that there may be now (beyond pure phenomenological justifications) really a compelling theoretical reason for any of them to gain one extra helicity freedom degree on getting massive. In the revised electroweak framework, the $\phi(x)$ isodoublet is clearly representable in two alternative manners, depending on whether the $\mathrm{SU(2)_L \otimes U(1)}_Y$ or $\mathrm{SU(2)_R \otimes U(1)}_{\bar{Y}}$ covariant picture is adopted. Recalling that the *effective* one-particle isospin operator is $\mathbf{T}$ in the former case and $-\mathbf{T}$ in the latter, we may put

$$\phi(x) = \phi_{\mathrm{L}}(x) \equiv \exp\left\{ i \left[\sum_{k=1}^{3} \theta_k(x) T_k + \eta(x) \right] \right\} \phi_\circ \tag{194}$$

and

$$\phi(x) = \phi_{\mathrm{R}}(x) \equiv \exp\left\{ i \left[\sum_{k=1}^{3} -\theta_k(x)(-T_k) + \eta(x) \right] \right\} \phi_\circ, \tag{195}$$

respectively. According to the former picture, the weak isospin acting on $\phi(x)$ is $\mathbf{T}^{\mathrm{w}} = \mathbf{T}$, and

$$T_3^{\mathrm{w}} \phi_\circ = T_3\, \phi_\circ = \left(-\frac{1}{2}\right) \phi_\circ, \qquad Y \phi_\circ = (+1)\, \phi_\circ\,; \tag{196}$$

whereas, according to the latter picture, the weak isospin acting on $\phi(x)$ is $\mathbf{T}^{\mathrm{w}} = -\mathbf{T}$, and

$$T_3^{\mathrm{w}} \phi_\circ = (-T_3)\, \phi_\circ = \left(+\frac{1}{2}\right) \phi_\circ, \qquad \bar{Y} \phi_\circ = (-1)\, \phi_\circ. \tag{197}$$

In the $\mathrm{SU(2)_L \otimes U(1)}_Y$ picture, the coupling (188) should be replaced by

$$\mathcal{L}^{(\mathrm{f})}_{\mathrm{Yukawa}} = \frac{1}{2} \sum_{f_i, \bar{f}_j} m_{f_i \bar{f}_j}\, \bar{\chi}^{(\mp)}_{f_i} \rho^{(\pm)}\, \chi_{\bar{f}_j} + \mathrm{H.c.} \tag{198}$$

($m_{f_i \bar{f}_j} = m_{ij}$) where f_i, $\bar{f}_j$ are *fermion* and *antifermion* flavor indices, respectively, and where the $\frac{1}{2}$ factor is due to the different normalizations of $\bar{\chi}^{(\mp)}_{f_i}$ and

$\chi_{\bar{f}_j}$ as compared with $\bar{\psi}_{i\mathrm{L}}^{(\mp)}$ and $\psi_{j\mathrm{R}}$. In the SU(2)$_\mathrm{R}$⊗U(1)$_{\bar{Y}}$ picture, equivalently, one should write

$$\mathcal{L}^{(\mathrm{f})}_{\mathrm{Yukawa}} = \frac{1}{2} \sum_{\bar{f}_i, f_j} m_{\bar{f}_i f_j}\, \bar{\chi}^{(\pm)}_{\bar{f}_i} \rho^{(\pm)} \chi_{f_j} + \mathrm{H.c.}, \tag{199}$$

where the inverted superscript $(\pm)$ for $\bar{\chi}_{\bar{f}_i}$ (as compared with $\bar{\chi}_{f_i}$) shows up either sign of the corresponding (opposite) eigenvalue of the weak-isospin third component. By use of the above unitarity-preserving procedure of approaching the extended unitary gauge, both (198) and (199) can be seen to yield just the same mass term: say,

$$\mathcal{L}^{(\mathrm{f})}_{\mathrm{mass}} = \frac{1}{2} \sum_{f_i, \bar{f}_j} m_{f_i \bar{f}_j}\, \bar{\chi}_{f_i}\, \chi_{\bar{f}_j} + \mathrm{H.c.}. \tag{200}$$

For a diagonal $m_{f_i \bar{f}_j}$ matrix with eigenvalues m_f, this reduces to

$$\mathcal{L}^{(\mathrm{f})}_{\mathrm{mass}} = \frac{1}{2} \sum_{f} m_f\, (\bar{\chi}_f\, \chi_{\bar{f}} + \bar{\chi}_{\bar{f}}\, \chi_f). \tag{201}$$

9. LINK BETWEEN THE WEINBERG MIXING AND THE ACTUAL SUPERSELECTION OF ELECTRIC-CHARGE EIGENSTATES

In the framework of the massless spin-$\frac{1}{2}$ particle model of Section 4, whether the SU(2)$_\mathrm{L}$⊗U(1)$_Y$ or SU(2)$_\mathrm{R}$⊗U(1)$_{\bar{Y}}$ picture is being adopted, the only conceivable scalar-charge current is the one of the whole "would-be-Dirac" type (109), with ψ $(= \psi\pm)$ being a chiral-field *mixture* like (76) and Q $(= q P_{\mathrm{in}})$ behaving like a one-particle charge operator as in (110). On passing to an $[\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y] \oplus [\mathrm{SU(2)_R} \otimes \mathrm{U(1)}_{\bar{Y}}]$ unified formalism – see the end of Section 6 – a corresponding "dressed" current can be built from (109), having an overall structure like

$$\bar{\psi}_+ \gamma^\mu Q\, \psi_+ \mathcal{P}_+ + \bar{\psi}_- \gamma^\mu Q\, \psi_- \mathcal{P}_- \quad (\psi_- = \gamma^5 \psi_+), \tag{202}$$

where $\mathcal{P}_+$ and $\mathcal{P}_-$ are the Casimir operators given by (162). Of course, as long as the two (fermion and antifermion) chiral fields χ_f and $\chi_{\bar{f}}$ are *massless* (so that either ψ_+ or ψ_- may only be a *mixture* of them), both the real fermion and the real antifermion so depicted are naturally bound to have *single* (and opposite) helicity eigenvalues and are stricly *prevented* from looking like actual scalar-charge eigenstates (except for the trivial case of a zero Q eigenvalue). This just corresponds to the fact that the scalar charge Q can never turn out to be diagonal (with nonzero eigenvalues) in the absence of mass. On the other hand, once mass is acquired, neither ψ_+ nor ψ_- will be bound any more to be a chiral-field mixture;

and so it will result that ψ_+ and ψ_- themselves have truly be made two (covariantly conjugated) Q eigenfields, each with one independent, unmeasurable phase.

In such a framework, the actual existence of nonzero-scalar-charge eigenstates is indeed a *non*trivial matter, closely related to the appearance of fermion masses. At variance with the standard approach to the electroweak model, one now has that *the Weinberg mixing looks just like an essential "superselective" mechanism for the electric charge, which further cannot be made working without the simultaneous generation of a mass, and the simultaneous gain of an extra helicity freedom degree, for the electrically charged fermion involved*. By adopting the above-seen, pure "vacuum" SSB model – with a *fourth* would-be-Goldstone boson in the place of the Higgs particle – one may consistently claim, on the other hand, just an *internal* origin for fermion-mass appearance, as a general effect implied by the emerging superselective attitude of a *scalar* variety of charges (as opposed to the *pseudoscalar* variety dominating the zero-mass primary stage). According to these nonstandard views, the Higgs doublet (as properly recast in terms of *four* would-be-Goldstone Hermitian fields) should now be thought of like a mere, *effective* "vacuum" field, which must so interact as to reproduce the *given* final spectrum of mass eigenstates. As is well-known, the spectrum in question consists of strict *flavor*-scalar-charge (besides electric-charge) eigenstates, with definite eigenvalues for *all* of the flavor scalar charges. Hence, in full agreement with what is apparently suggested by a phenomenological approach, just the combined internal contributions of both the electric charge and the flavor scalar charges should ultimately underlie such a spectrum, with no more room for the conjecture of some "external" real agent at the origin of it (note that strictly speaking, a special contribution, the same for all quarks, should also come from the superselection of color eigenstates as opposed to anticolor ones).

In general, once a physical pair of mutually conjugated scalar-charge eigenfields is superselected (at the price of giving *mass* to both the particle and the antiparticle), one gets

$$\psi_+ \ (\bar{\psi}_+) \rightarrow \psi_f \ (\bar{\psi}_f), \quad \psi_- \ (\bar{\psi}_-) \rightarrow \psi_{\bar{f}} \ (\bar{\psi}_{\bar{f}}) \tag{203}$$

as well as, recalling Eqs. (164) and (165),

$$\mathcal{P}_+ \rightarrow |f\rangle\langle f|, \quad \mathcal{P}_- \rightarrow |\bar{f}\rangle\langle \bar{f}|\,; \tag{204}$$

and by substituting (110), the current (202) is at last turned into (62), with Q as given by (110) being made just a representation of the scalar-charge operator (61).

10. *P* AND *C* BREAKINGS DUE TO THE WEINBERG MIXING, AND *P*-INVARIANCE SURVIVAL FOR THE CHARGED-CURRENT COUPLINGS IN THE PRESENCE OF *CP* VIOLATION

Herein the fermion-mass Lagrangian term is given by the (diagonalized) general formula (201), equally valid for both $SU(2)_L \otimes U(1)_Y$ and $SU(2)_R \otimes U(1)_{\bar{Y}}$ alternative pictures. Any chiral-field pair χ_f, $\chi_{\bar{f}}$ appearing in (201) is no longer *strictly* a pair of mass eigenfields as in the zero-mass case. Indeed, it is the two corresponding linear combinations defined in (76) that have been made mass eigenfields (with *opposite* eingenvalues): if (201) is rewritten in terms of each actual mass eigenfield ψ_f and/or $\psi_{\bar{f}}$ $(= \gamma^5 \psi_f)$ as given by (25), it then becomes

$$\mathcal{L}^{(f)}_{\text{mass}} = \frac{1}{2} \sum_f \left[m_f \bar{\psi}_f \psi_f + (-m_f) \bar{\psi}_{\bar{f}} \psi_{\bar{f}} \right] = \sum_f m_f \bar{\psi}_f \psi_f = \sum_f (-m_f) \bar{\psi}_{\bar{f}} \psi_{\bar{f}} \tag{205}$$

or, by use of an $[SU(2)_L \otimes U(1)_Y] \oplus [SU(2)_R \otimes U(1)_{\bar{Y}}]$ unified formalism,

$$\mathcal{L}^{(f)}_{\text{mass}} = \sum_f \left[m_f \, \bar{\psi}_f \psi_f \, \mathcal{P}_+ + (-m_f) \, \bar{\psi}_{\bar{f}} \, \psi_{\bar{f}} \, \mathcal{P}_- \right], \tag{206}$$

where it may directly be put $\mathcal{P}_+ = |\, f \rangle \langle f \,|$ and $\mathcal{P}_- = |\, \bar{f} \rangle \langle \bar{f} \,|$. On the other hand, despite the appearance of fermion masses and the superselection of scalar-charge eigenstates, a surviving presence of chiral fields will still be dynamically experienced in weak-isospin couplings. This, by an inspection of (117), may consistently be related to the fact that the $SU(2)_L$ generator, $\mathbf{T}_L$, in the $SU(2)_L \otimes U(1)_Y$ picture, as well as the $SU(2)_R$ generator, $\mathbf{T}_R$, in the $SU(2)_R \otimes U(1)_{\bar{Y}}$ picture, are themselves able to keep each χ_f (fermion) and $\chi_{\bar{f}}$ (antifermion) field *superselected*, respectively.

Like the starting scheme (without masses), the final scheme (with masses) will include two equivalent covariant varieties of gauge couplings, which may only alternately be utilized, according to whether the $SU(2)_L \otimes U(1)_Y$ or $SU(2)_R \otimes U(1)_{\bar{Y}}$ symmetry group is being adopted. Yet, strictly speaking, these two available electroweak formulations should together be taken as mere complementary aspects of a "whole" formulation symmetrically dealing with fermions as well as antifermions. This is because, in a rigorous one-particle covariant approach, the electromagnetic U(1) gauge group has now a generator being a *charge operator* of the type

$$Q(q) = q \, (\mathcal{P}_+ - \mathcal{P}_-) \tag{207}$$

(q and $-q$ generally denoting the given, particle and antiparticle, electric-charge eigenvalues involved), so that it can only be a subgroup of the *overall*, left–right symmetric group (169), and not of either single group $SU(2)_L \otimes U(1)_Y$ or $SU(2)_R \otimes U(1)_{\bar{Y}}$.

The general electroweak-gauge-Lagrangian pattern (the IVB electromagnetic coupling sector apart) can just be obtained by recasting (170) in terms of the final neutral spin-1 mass eigenfields given by (186). It now consists of a "whole" L + R Lagrangian of the type

$$\mathcal{L}_{\text{int(L+R)}} = \left[\, \mathcal{L}^{\text{w}}_{\text{int(L)}} - q\, \bar{\psi}_f \gamma^\mu \psi_f\, A_\mu \,\right] \mathcal{P}_+ + \left[\, \mathcal{L}^{\text{w}}_{\text{int(R)}} - (-q)\, \bar{\psi}_{\bar{f}}\, \gamma^\mu \psi_{\bar{f}}\, A_\mu \,\right] \mathcal{P}_-, \tag{208}$$

where (by a direct use of a manifestly covariant form) one has

$$\mathcal{L}^{\text{w}}_{\text{int(L)}} = -2^{-3/2} g \left(\bar{D}_\text{L} \gamma^\mu\, T^{\text{w}+} D_\text{L}\, W_\mu + \bar{D}_\text{L} \gamma^\mu\, T^{\text{w}-} D_\text{L}\, W^\dagger_\mu \right) - \left(\frac{g}{2\cos\theta}\, \bar{D}_\text{L} \gamma^\mu\, T^\text{w}_3 D_\text{L} - q \tan\theta\, \bar{\psi}_f \gamma^\mu \psi_f \right) Z_\mu \tag{209}$$

$$\mathcal{L}^{\text{w}}_{\text{int(R)}} = -2^{-3/2} g \,(\bar{D}_\text{R} \gamma^\mu\, T^{\text{w}+} D_\text{R}\, W_\mu + \bar{D}_\text{R} \gamma^\mu\, T^{\text{w}-} D_\text{R}\, W^\dagger_\mu) - \left[\frac{g}{2\cos\theta}\, \bar{D}_\text{R} \gamma^\mu\, T^\text{w}_3 D_\text{R} - (-q) \tan\theta\, \bar{\psi}_{\bar{f}}\, \gamma^\mu \psi_{\bar{f}} \right] Z_\mu \tag{210}$$

($\theta = \theta_\text{W}$; $T^{\text{w}\pm} = T^\text{w}_1 \pm i\, T^\text{w}_2$; $\sqrt{2} W_\mu = W_{1\mu} - i\, W_{2\mu}$) and where, according to the formalism of Secs. 2, 5 and *at variance with the* $V - A$ *theory*, either D_L or D_R is left a strict *chiral-field* isodoublet, with the fields normalized as in (24). Another nonstandard covariant feature of (208) comes from (115): now the three components $T^{\text{w}\pm}$, T^w_3 of the one-particle weak isospin $\mathbf{T}^\text{w}$ are also individually defined as "pseudoscalar-charge operators." Note that the original explicit presence of (right-handed) *antifermion* fields in the $\mathcal{P}_+$ sector, as well as of (left-handed) *fermion* fields in the $\mathcal{P}_-$ sector, have automatically disappeared as a consequence of the Weinberg mixing: the former sector now provides a genuine (positive- and negative-energy) *fermion* dynamical picture, just in terms of ψ_f and χ_f fields, while the latter a genuine (positive- and negative-energy) *antifermion* dynamical picture, just in terms of $\psi_{\bar{f}}$ and $\chi_{\bar{f}}$ fields ("covariantly conjugated" to the ψ_f and χ_f ones). Formula (208) includes the "whole" *fermion-plus-antifermion covariant electromagnetic gauge coupling*

$$-Q(q)\, (\bar{\psi}_f \gamma^\mu \psi_f\, \mathcal{P}_+ + \bar{\psi}_{\bar{f}}\, \gamma^\mu \psi_{\bar{f}}\, \mathcal{P}_-)\, A_\mu, \tag{211}$$

which merges the two equivalent couplings in turn available according to the picture chosen. In (211), the electric charge (operator) $Q(q)$, given by (207), is more specifically expressible – with the help of Eqs. (133) and (134), and of Eqs. (145) and (146) – as

$$Q = [(t_{3\text{L}} + y/2)\, |e|\,]\, \mathcal{P}_+ \;+\; [(t_{3\text{R}} + \bar{y}/2)\, |e|\,]\, \mathcal{P}_-. \tag{212}$$

Also, note that actually, $\bar{\psi}_f \gamma^\mu \psi_f = \bar{\psi}_{\bar{f}} \gamma^\mu \psi_{\bar{f}}$, (with $\psi_{\bar{f}} = \gamma^5 \psi_f$ and $\bar{\psi}_{\bar{f}} = -\bar{\psi}_f \gamma^5$). This, of course, is quite admissible on the basis of what has been pointed out in Sections 2, 3. The fact is that the two available, *fermionic* and *antifermionic*, pictures above are self-contained and *mutually exclusive* (as is formally ensured by the presence of their respective Casimir operators $\mathcal{P}_+$ and $\mathcal{P}_-$). In the overall framework embodying them both, one has that every fermion field ψ_f or χ_f (belonging to the former picture) and corresponding antifermion field $\psi_{\bar{f}}$ or $\chi_{\bar{f}}$ (belonging to the latter picture) are *covariantly* conjugated to each other and share *identical* "particle" annihilation operators as well as *identical* "hole" creation operators (with no "particle" $\rightleftharpoons$ "hole" interchange on passing from one to the other picture): what just happens is that the former description associates "particle" with fermion (and "hole" with antifermion) while the latter associates "particle" with antifermion (and "hole" with fermion). Quite a similar remark applies to the two charged boson fields W_μ and $W_\mu^\dagger$, which do not appear interchanged, either, on passing from one to the other picture: the point is merely that W_μ ($W_\mu^\dagger$) will annihilate a "particle" ("antiparticle") coinciding with W^+ (W^-) in the former picture, and with W^- (W^+) in the latter.

The basic new peculiar feature common to all final charged-current couplings (in both lepton and quark sectors) is that they do *naturally* reproduce the "maximal parity-violation" effect, in a way, i.e., neither being *ad hoc* imposed nor involving any *real* failure of P symmetry itself. This holds even in the presence of a (standardly parametrized) *CP* violation (Kobayashi and Maskawa, 1973), thanks to the fact that according to (25), "chiral fields" (with *fixed*, and opposite, chiralities for fermions and antifermions) are now *regular* fields just like Dirac ones, and the parity matrix, γ^0, covariantly applies to them just the same as to Dirac fields. Since herein the whole *CP* operation turns out still to be defined as in the $V - A$ formalism, it then follows, by applying the *CPT* theorem, that the observed *CP* failure can no longer be said to affect the symmetry under CT (T being the ordinary time reversal): rather, it would now amount to a breaking of both C and T individual symmetries, but *not* of CT symmetry, and its origin should now be simply ascribed to *those flavor scalar charges involved in the quark-family triplication*. In this regard, note that such a view on the question of the origin of *CP* violation is fully consistent with the well-known fact that no *CP* violation can in principle occur without the existence of at least *three* quark families. Within the leptonic sector (where mass eigenstates may be assumed to coincide with gauge eigenstates) the charged-current couplings are now consistently obeying C symmetry, too: this may clearly happen owing to their actual dependence on *no longer fictitious* chiral fields, which also under C (and not only under P) are transformed just like Dirac fields. On the contrary, neither P nor C symmetry is still generally obeyed by the final (lepton and quark) neutral-current weak couplings, even though as a mere result of the *interference* between scalar- and pseudoscalar-charge dynamics that is now induced by the Weinberg mixing: for either an electrically charged lepton or a quark, the final neutral weak current (as taken in its antisymmetrized version) is here made up of an *axial-vector* (i.e., the original weak-isospin neutral current) plus a *vector* (i.e., the electromagnetic current) which are (the former) left unchanged and (the latter) reversed by C. Symmetry under CP can obviously be ensured for such couplings, by setting

$$CPZ_{\mu}(x_{\nu})\, P^{\dagger}C^{\dagger} = Z^{\mu}(x^{\nu}). \tag{213}$$

Since A_{μ}, in its turn, is a vector inverted by C, one thus finds – also in view of (156) – that the gauge-field transformation (186) can be covariant only under CP (and not under P and C separately).

A few words should further be spent, finally, as to the strictly internal, new operation $C_{\text{cov}}P_{\text{in}}$, with C_{cov} defined by Eqs. (80) and (164), and P_{in} defined by Eqs. (79) and (165). It stands herein for the *total* "covariant charge-conjugation" operation, which applies to both *scalar* and *pseudoscalar* charge varieties (together involved in the new electroweak model) and interchanges fermions and antifermions as globally taken in their own *dual* – either "Dirac" or "chiral" – nature. By the way, note that C_{cov} alone acts on the electromagnetic sector (211) just the same as according to the prescription (65). The pure gauge Lagrangian pattern (208) is still invariant under $C_{\text{cov}}P_{\text{in}}$ as before the Weinberg mixing (recall that A_{μ} and Z_{μ} are linear combinations of two gauge fields that are both supposed to be left unchanged by $C_{\text{cov}}P_{\text{in}}$): the effect of $C_{\text{cov}}P_{\text{in}}$ will just be to transform all the single (charged current, neutral current, and electromagnetic) $\mathcal{P}_{+}$ and $\mathcal{P}_{-}$ subsectors into each other. Such a symmetry appears to be lost (at least, for the charged-current subsectors) in the case of the quark-family triplication, once an explicit reference is made to the actual quark mass eigenfields. Therein the Kobayashi–Maskawa matrix that should mix the three replicas of the $\mathcal{P}_{-}$ (antiquark) sector is clearly the *complex conjugate* of the one already mixing the three replicas of the $\mathcal{P}_{+}$ (quark) sector; and this cannot surely be obtained by applying a mere linear operation (like $C_{\text{cov}}P_{\text{in}}$) to the $\mathcal{P}_{+}$ sector.

11. ON THE RECOVERED *P* AND *C* SYMMETRIES IN THE *CP*-CONSERVING VARIETY OF THE CHARGED-CURRENT WEAK COUPLINGS

For a full understanding of the symmetry retrievals in the charged-current sector of (208), the basic point to bear in mind is that the weak-isospin components are herein *pseudoscalar* quantities, which furthermore *anticommute* with any scalar charges. As an immediate result, weak-isospin eigenstates are now strictly predicted to behave *as if carrying scalar charges definite only in magnitude and maximally indefinite in sign*. Such a property can at most allow a *dual* – either

"Dirac" or "chiral" – behavior of quarks and electrically charged leptons, which is just evidenced by the alternate presence of Dirac fields (of the ψ_f type) and *true* chiral fields (of the χ_f type) in most of the final couplings: while a "Dirac" fermion clearly looks like a pure scalar-charge eigenstate, which may be said to have a maximally uncertain "chirality" and a certain (relative) intrinsic parity, a "chiral" fermion looks instead like a pure pseudoscalar-charge eigenstate, which may conversely be said to have a certain "chirality" and a maximally uncertain (relative) intrinsic parity. Of course, a *dual* nature (still due to the coexistence of two anticommuting, pseudoscalar and scalar, charge varieties) should also be acknowledged to the charged gauge bosons $W^{\pm}$, as they are similarly predicted to behave either like pure pseudoscalar-charge eigenstates, in the weak couplings, or like pure scalar-charge eigenstates, in the electromagnetic couplings.

Within this general framework, let a "chiral" particle be more widely standing for any (whether fermion or boson) particle on its appearing as a pseudoscalar-charge eigenstate. Then, the recovered P-invariance property in every charged-current weak coupling will simply mean that *the mere space-inverted image of a "chiral" particle does already represent a "chiral" antiparticle, with no need of further applying a charge-conjugation operation*. It should however be emphasized that such a sort of P symmetry, like the more familiar one experienced in scalar-charge dynamics, cannot in general correspond but to a *partial* view of the particle nature, even though that view itself, once it apparently enters into play as the only allowable one, may just be taken as effectively representing the particle *in toto*. For instance, let $\chi_f(x^\mu)$ be the field of a (positive and negative energy) "chiral" fermion which is involved in a charged-current weak coupling belonging to the $\mathcal{P}_+$ sector of (208), and let $\chi_{\bar{f}}(x^\mu)$ be the pure chirality-conjugate of $\chi_f(x^\mu)$, associated with the corresponding (positive and negative energy) "chiral" antifermion in the $\mathcal{P}_-$ sector. One and the same *complete* standard Fock space, $\mathcal{F}^\circ$, will be alternately available for the fermion and the antifermion in question (depending on whether the $\mathcal{P}_+$ or $\mathcal{P}_-$ picture is being adopted). Under space inversion, $\chi_f(x^\mu)$ is transformed as follows:

$$\chi_f(x^\mu) \longrightarrow \chi_f^{(P)}(x_\mu) = P_{\text{ex}}^\dagger \, \chi_{\bar{f}}\,(x_\mu) P_{\text{ex}} = \gamma^0 \chi_f(x^\mu) \tag{214}$$

($P = P_{\text{ex}} P_{\text{in}} = P_{\text{in}} P_{\text{ex}}$), and quite an analogous transformation holds for $\chi_{\bar{f}}(x^\mu)$. According to (214), P has not only the usual "external" action in $\mathcal{F}^\circ$, but also an "internal" one (of chirality inversion, $\chi_f \to \chi_{\bar{f}}$) in the two-dimensional space spanned by the field pair $(\chi_f, \chi_{\bar{f}})$; and the P-transformed field, $P_{\text{ex}}^\dagger \, \chi_{\bar{f}}\,(x_\mu) P_{\text{ex}}$, correspondingly describes the space-inverted kinematical state of a P-transformed "chiral" fermion just coinciding with the "chiral" antifermion. Note, on the other hand, that if P is applied to the "Dirac"-fermion field $\psi_f(x^\mu) = 2^{-1/2}[\chi_f(x^\mu) + \chi_{\bar{f}}(x^\mu)]$, dual to $\chi_f(x^\mu)$, then the whole action of P is automatically reduced to the pure "external" one:

$$\psi_f(x^\mu) \longrightarrow \psi_f^{(P)}(x_\mu) = \psi_f^{(P_{\rm ex})}(x_\mu) = \gamma^0 \psi_f(x^\mu). \tag{215}$$

So, the P-invariance property for the charged-current couplings in the $\mathcal{P}_+$ sector of (208) should typically signify that they (which are referred to positive- and negative-energy *fermions*) can just as well work as couplings being referred to positive- and negative-energy *antifermions* in the space-inverted frame. This should also imply an interchange of the original physical roles assigned to W_μ and $W_\mu^\dagger$: if the gauge field W_μ ($W_\mu^\dagger$) is assumed to annihilate an intermediate vector boson W^+ (W^-) then its P-counterpart should vice versa be assumed to annihilate an intermediate vector boson W^- (W^+). Under such special dynamical circumstances, P alone may be said to have the additional effect of a "covariant charge-conjugation" operation just simulating a real change of a particle into an antiparticle. In more specific terms, since no "particle" $\rightleftharpoons$ "hole" conjugation is induced by P on fermion fields, what is truly involved is the passage from a complete ("fermionic") picture where "particle" = fermion (and "hole" = antifermion) to a complete ("antifermionic") picture where "particle" = antifermion (and "hole" = fermion). Of course, except for the zero-mass case, the mere parity P cannot really be taken as a *true* particle $\rightleftharpoons$ antiparticle conjugation operation: for example, a "Dirac" particle (typically behaving like a pure scalar-charge eigenstate) is not turned at all by P into a "Dirac" antiparticle. The same can be said for C, but in a reverse manner: C has, on "chiral" particles, no longer the effect of truly changing a particle into an antiparticle, even though such an effect is still regularly present when C is applied to "Dirac" particles. This can be inferred from the fact that C now yields, e.g.,

$$\chi_f \;\longrightarrow\; \chi_f^{(C)} = C_{\rm st}^\dagger\, \chi_{\bar f}\, C_{\rm st} = C \widetilde{\chi}_f^\dagger \tag{216}$$

besides

$$\psi_f \;\longrightarrow\; \psi_f^{(C)} = \psi_f^{(C_{\rm st})} = C \widetilde{\psi}_f^\dagger \tag{217}$$

($C_{\rm st}$ denoting the C operation as just defined in $\mathcal{F}^\circ$, and C being here used to denote the C-matrix as well). The point is that C, like P, has also an action (of chirality inversion) in the $(\chi_f, \chi_{\bar f})$ space (i.e., $C = C_{\rm st} P_{\rm in} = P_{\rm in} C_{\rm st}$): the net effect of C will be thus to transform a "chiral" fermion taken as a "particle" (in a picture where "particle" = fermion and "hole" = antifermion) into *itself* taken as a "hole" (in a *new* picture where "particle" = antifermion and "hole" = fermion) and to interchange not only W_μ and $W_\mu^\dagger$, but also their assigned roles of annihilation operators for W^+ and W^- bosons. Identical conclusions can clearly be drawn, even when reference is made to the alternative (antifermionic) charged-current couplings in the $\mathcal{P}_-$ sector of (208). It is worth, nevertheless, stressing the fact that if the new electroweak model is globally taken in its *pseudoscalar*- plus *scalar*-charge dynamical contributions, then only *CP as a whole* is left a true particle $\rightleftharpoons$ antiparticle conjugation operation.

For a more straightforward evaluation of both P and C individual effects on "chiral"-particle systems, it turns out convenient to make an explicit use of a normally ordered formalism. First it should be borne in mind that generally, given a gauge coupling term like $q^{\rm ch}\bar{\chi}_{\rm b}\gamma^{\mu}\chi_{\rm a}W_{\mu}$, with $q^{\rm ch}$ being a (pseudoscalar) charge eigenvalue which under P (or $P_{\rm in}$) is taken into $-q^{\rm ch}$, the new way (33) of applying the C-matrix enables one to write

$$q^{\rm ch}[(:\bar{\chi}_{\rm b}\gamma^{\mu}\chi_{\rm a}W_{\mu}:)+{\rm H.c.}]=\frac{q^{\rm ch}}{2}\left[\left(\bar{\chi}_{\rm b}\gamma^{\mu}\chi_{\rm a}-\bar{\chi}_{\rm a}^{(C)}\gamma^{\mu}\chi_{\rm b}^{(C)}\right)W_{\mu}+{\rm H.c.}\right] \quad (218)$$

or (after appropriate rearrangements involving also the Hermitian conjugate term)

$$q^{\rm ch}[(:\bar{\chi}_{\rm b}\gamma^{\mu}\chi_{\rm a}W_{\mu}:)+{\rm H.c.}]=\frac{q^{\rm ch}}{2}\left[\left(\bar{\chi}_{\rm b}\gamma^{\mu}\chi_{\rm a}W_{\mu}-\bar{\chi}_{\rm b}^{(C)}\gamma^{\mu}\chi_{\rm a}^{(C)}W_{\mu}^{\dagger}\right)+{\rm H.c.}\right]. \quad (219)$$

A glance at the expression within square brackets in (219) then shows that the action of the (covariant) transformation

$$P_{\rm in}:\begin{cases}\chi_{\rm a}\to\chi_{\bar{\rm a}}, & \bar{\chi}_{\rm b}\to\bar{\chi}_{\bar{\rm b}}, \quad \chi_{\rm a}^{(C)}\to\chi_{\bar{\rm a}}^{(C)}, \quad \bar{\chi}_{\rm b}^{(C)}\to\bar{\chi}_{\bar{\rm b}}^{(C)},\\ W_{\mu}\to W_{\mu}, & W_{\mu}^{\dagger}\to W_{\mu}^{\dagger}, \quad q^{\rm ch}\to -q^{\rm ch},\end{cases} \quad (220)$$

with $\chi_{\bar{\rm a}}=[\chi_{\rm a}^{(C)}]^{(C_{\rm st})}$, and so on, can equally well be reproduced (via a noncovariant, *effective* approach) by making instead the substitutions

$$\begin{cases}\chi_{\rm a}\to\chi_{\rm a}^{(C_{\rm st})}, & \bar{\chi}_{\rm b}\to\bar{\chi}_{\rm b}^{(C_{\rm st})}, \quad \chi_{\rm a}^{(C)}\to\chi_{\rm a}^{(C)(C_{\rm st})}, \quad \bar{\chi}_{\rm b}^{(C)}\to\bar{\chi}_{\rm b}^{(C)(C_{\rm st})},\\ W_{\mu}\rightleftharpoons W_{\mu}^{\dagger}. \end{cases} \quad (221)$$

Such an approach – which has the advantage of always dealing (as usual) with one and the same "particle"–"antiparticle" formal picture – leads indeed to the self-explaining *effective* equalities $P_{\rm in}=C_{\rm st}$, $P=(CP)_{\rm st}$ $(=CP)$ and $C=1$. Of course, these equalities may actually be taken into account only if the gauge eigenfields $\chi_{\rm a,b}$ are also *mass* eigenfields (as it occurs for the CP-conserving sector of the charged-current weak couplings).

12. CONCLUDING REMARKS

A new version of the whole electroweak theory has been here worked out, which naturally embodies (regardless of experience) the so-called "maximally P-violating" phenomenology and is furthermore able to provide a basic explanation of it in full observance of mirror symmetry. The formalism underlying the proposed scheme is a covariant fermion–antifermion generalization of the usual relativistic quantum field formalism for massive spin-$\frac{1}{2}$ fermions. It strictly admits not only Dirac fields, as eigenfields of *scalar* charges, but also true "chiral fields," as eigenfields of *pseudoscalar* charges, and correspondingly supports a *dual* model of a massive point fermion, characterized by the internal coexistence of two *anticommuting* (scalar and pseudoscalar) charge varieties. According to such a model, there would be two complementary and *mutually exclusive* – "Dirac" and "chiral" – natures inside any single real quark and electrically charged lepton, whose alternation should just depend on whether a pure scalar- or pseudoscalar-charge dynamics is working (an actual interference between these two natures would be experienced only in the neutral-current electroweak sector, as a result both of the Weinberg mixing and of the overlapping of electromagnetic and weak contributions). The role of pseudoscalar charges should in particular be expected to become really crucial and predominant in the zero-mass limit: the reason is because a simple *two-component* fermion theory is thus spontaneously come to, in which a massless spin-$\frac{1}{2}$ fermion and its antiparticle are universally remodelled as two sheer "chiral" particles, always looking like *pseudoscalar*-charge eigenstates and being the mere *mirror* (or *helicity-conjugate*) counterparts of each other. This theory – which naturally deals with an *only left-handed* massless fermion and an *only right-handed* massless antifermion without breaking parity symmetry – is just the one underlying the new, both P- and C-invariant, zero-mass primary version of the electroweak scheme. As an immediate physical consequence, one now has that a massless spin-$\frac{1}{2}$ fermion may at most bear scalar (additional) charges *maximally uncertain in sign*, and that only by acquiring a *mass*, and by gaining an *extra* helicity freedom degree, may it also be enabled to appear as a "Dirac" particle (with sharp scalar-charge eigenvalues). A strict "internal" motivation is thus found for the appearance of fermion masses, which furthermore requires a modified mechanism of fermion-mass generation: what should now be involved is the simple absorption of a (fourth) would-be-Goldstone boson (just needed to counterbalance the actual gain, for the fermion, of the *originally missing* helicity freedom degree). Another basic feature that deserves to be mentioned is the fact that the recovered P-invariance property of the pure weak-isospin gauge couplings is merely "hidden" by the Weinberg mixing and still holds in the charged-current sector even in the presence of a (standardly parametrized) CP violation: the experienced CP failure may now be reduced to a pure C failure, to which there should correspond a T (i.e., time reversal) violation being such that CT itself, and not only CPT, is left a symmetry operation.

In short, as compared with the standard version, the proposed electroweak scheme comes to the following main new results:

(1) It is able to predict the "maximal P-violation" effect without the help of any *ad hoc* prescription and to replace the (phenomenological) Dirac-field "$V - A$" formalism with a natural, theoretically well-grounded, chiral-field "V" formalism.

(2) It can also give an answer as to the *origin* of the "maximally P-violating" phenomenology, on the basis of a general, pure theoretic approach (in terms of two *anticommuting*, scalar and pseudoscalar, charge varieties) which paradoxically recovers both P and C individual symmetries.
(3) It rigorously establishes, beyond experimental data, that *no* right-handed massless neutrinos (left-handed massless antineutrinos) may ever exist.
(4) It converts the (ad-hoc conjectured) standard "right-handed fermion" isosinglets into (natural) *antifermion* $SU(2)_L$ isosinglets, thus resolving the usual apparent final inconsistency due to the fact that two mere "chiral projections" of one and the same (Dirac) massive fermion field would find themselves belonging to two different weak-isospin representations.
(5) It provides a deep theoretical motivation for the appearance of fermion masses, as just a step absolutely necessary to obtain *superselected* scalar-charge (and primarily, electric-charge) eigenstates.
(6) It demands, for either an electrically charged lepton or a quark, a mass-generating mechanism being just like the one for an intermediate vector boson and no longer involving the Higgs particle (this is clearly related to the fact that even a massless spin-$\frac{1}{2}$ fermion, and not only a massless spin-1 boson, has now to gain an *extra* helicity freedom degree in order to be made massive).

The pure "vacuum" Higgs mechanism here adopted (involving *four* would-be-Goldstone bosons and nothing else) has been shown not to spoil the renormalizability and unitarity of the theory; it has also been proved to be a *self-consistent* mechanism, in the sense that the natural, strict absence of any self-coupling term inside the new Higgs Lagrangian (180) is by itself able to ensure the inapplicability of the well-known standard argument (Cheng and Li, 1984) against a number of more than *three* would-be-Goldstone bosons in all.

REFERENCES

Aitchison, I. J. R. and Hey, A. J. G. (1984). *Gauge Theories in Particle Physics*, Adam Hilger, Bristol, pp. 11–16.

Ambler, E., Hayward, R. W., Hoppes, D. D., Hudson, R. R., and Wu, C. S. (1957). *Physical Review* **105**, 1413.

Barut, A. O. (1972). *Physics Letters B* **38**, 97.

Barut, A. O. (1973). *Physics Letters B* **46**, 81.

Barut, A. O. (1982). *Foundations of Physics* **13**, 7.

Barut, A. O. and Ziino, G. (1993). *Modern Physics Letters A* **8**, 1011.

Cheng, T. P. and Li, L. F. (1984). *Gauge Theory of Elementary Particle Physics*, Oxford University Press, New York, p. 246.

Costa de Beauregard, O. (1982). *Foundations of Physics* **12**, 861.

Costa de Beauregard, O. (1984). In *The Wave-Particle Dualism*, S. Diner *et al*. eds. D. Reidel, p. 485.
Feynman, R. P. (1949). *Physical Review* **76**, 769.
Feynman, R. P. and Gell-Mann, M. (1958). *Physical Review* **109**, 193.
Glashow, S. L. (1961). *Nuclear Physics* **22**, 579.
Goldstone, J. (1961). *Nuovo Cimento* **19**, 154
Goldstone, J., Salam, A., and Weinberg, S. (1962). *Physical Review* **127**, 965.
Halzen, F. and Martin, A. D. (1984). *Quarks and Leptons*, J. Wiley & Sons.
Kobayashi, M. and Maskawa, M. (1973). *Progress of Theoretical Physics* **49**, 652.
Landau, L. D. (1957). *Nuclear Physics* **3**, 127.
Lee, T. D. and Yang, C. N. (1956). *Physical Review* **104**, 254.
Lee, T. D. and Yang, C. N. (1957). *Physical Review* **105**, 1671.
Majorana, E. (1937). *Nuovo Cimento* **14**, 171.
Marshak, R. E. and Sudarshan, E. C. G. (1958). *Physical Review* **109**, 1860.
Mohapatra, R. N. and Pati, J. C. (1975). *Physical Review D* **11**, 566.
Narlikar, J. V. and Padmanabhan, T. (1986). *Gravity, Gauge Theories and Quantum Cosmology*, D. Reidel, p. 129.
Recami, E. and Ziino, G. (1976). *Nuovo Cimento A* **33**, 205.
Ross, G. G. (1993). *Contemporary Physics* **34**, 79.
Sakurai, J. J. (1958). *Nuovo Cimento* **7**, 649.
Salam, A. (1957). *Nuovo Cimento* **5**, 299.
Salam, A. (1968). In *Elementary Particle Theory*, N. Svartholm ed. Almquist and Wiksell, Stockholm, p. 367.
Stüeckelberg, E. C. G. (1948). *Physical Review* **74**, 218.
Tiomno, J. (1955). *Nuovo Cimento* **1**, 226.
Weinberg, S. (1967). *Physical Review Letters* **19**, 1264.
Weyl, H. (1929). *Zeitschrift fur̈ Physik* **56**, 330.
Ziino, G. (1996). *International Journal of Modern Physics A* **11**, 2081.
Ziino, G. (2000). *International Journal of Theoretical Physics* **39**, 51.
Ziino, G. (2003). *Hadronic Journal* **26**, 655.